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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS (2009-10)
Course: M.Sc. MATHEMATICS (Final Year)

Note: *Students are advised to read the separate enclosed instructions before beginning the writing of assignments.*

Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme/ Practical classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

ANSWER ALL TOPICS

Paper-V: Complex Analysis

- 1) a) Given three numbers z_1, z_2, z_3 such that $|z_1| = |z_2| = |z_3| = r$ and $z_1 + z_2 + z_3 = 0$. Show that these complex numbers are vertices of an equilateral triangle inscribed in the radius of r centred at origin.
- b) Define a function $f(z) = \sqrt{|xy|}$. Show that it satisfies Cauchy Riemann equations at $(0, 0)$ but the function is not differentiable at that point.
- 2) a) If $f(z)$ is analytic and $|f(z)| \leq M$ for $|z| < R$, find an upper bound for $|f^{(n)}(z)|$ in $|z| \leq r < R$.
- b) If $f(z)$ is analytic and $\text{Im } f(z) \geq 0$ for $\text{Im } z > 0$, Show that

$$\frac{|f(z) - f(z_0)|}{|f(z) - \overline{f(z_0)}|} \leq \frac{|z - z_0|}{|z - \overline{z_0}|} \quad \text{And} \quad \frac{|f'(z)|}{\text{Im } f(z)} \leq \frac{1}{y} \quad (z = x + iy)$$

- 3) a) State and prove Rouché's theorem. Further deduce that fundamental theorem of Algebra.
- b) Evaluate the following integral by the method of residues:

$$\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx, \text{ a real.}$$

Paper-VI: Topology

- 1) a) Define Hausdorff space. Prove that every finite set in a Hausdorff space is closed.
- b) Show that X is Hausdorff if and only if the diagonal $\Delta = \{x \times x \mid x \in X\}$ is closed in $X \times X$.
- 2) Show that
 - a) A locally compact Hausdorff space is completely regular.
 - b) Every locally compact Hausdorff space is regular.
- 3) a) Show that a closed subspace of a normal space is normal.
- b) If X and Y are normal, $X \times Y$ may not be normal.

Paper-VII: Real and Functional Analysis

1. a) Construct an uncountable set of measure zero.
- b) Find the Dini derivatives of $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0$.
- c) Define absolute continuous and a bounded variation. Is \sqrt{x} absolutely continuous on $[0, 1]$? Further show that $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & 0 \leq x \leq 1 \\ 0 & x = 0 \end{cases}$ is of bounded variation on $[0, 1]$.
2. a) If $\int_a^b f < \infty$ and $F(x) = F(a) + \int_a^x f(t)dt$, Show $F' = f$ a.e., on $[a, b]$. (Prove all lemmas you may use).
- b) If S is the set of all continuous functions on closed interval $[a, b]$ and d is a metric on S defined by $d(x, y) = \int_a^b |x(t) - y(t)|dt$, then show that S is not complete.
- c) Prove there are continuous functions on $[0, 1]$ which are nowhere differentiable.
3. a) Let X be metric space, U_1 and U_2 open and dense in X . show $U_1 \cap U_2$ is dense in X . If X is complete and $\{U_n\}_{n=1}^{\infty}$ is a sequence of dense open sets in X . Show $\bigcap_{n=1}^{\infty} U_n$ is dense in X . Show Q_1 is of 1st category and $R' - Q$ is of 2nd category.
- b) If the dual X' of a normed linear space X is separable, show X is separable. Is the converse true? What if X is reflexive?
- c) If M is a closed linear subspace of a normed linear space X and x_0 is a vector not in X , then prove that there exists a functional f in X^* such that $f(M) = 0$ and $f(x_0) \neq 0$.

Paper-VIII: Numerical Analysis

1. Extract a quadratic factor of the form $x^2 - px - q$ from a polynomial $x^5 - 2x^4 + 4x^3 - x^2 - 7x + 5 = 0$, choose $p = 1, q = 1$, Perform 3 iterations.
2. Obtain the cubic spline interpolation polynomial for the following data

x	0	1	2	3	4
f(x)	-1	0	7	26	63

Given $S''(0) = S''(4) = 0$

3. Solve an Initial value problem $y' = \sin(x) - y^2, y(0) = 1$ using Adam's Predictor-Corrector method.
