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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS (2009-10)
Course: **M.Sc. MATHEMATICS** (Previous)

Note: *Students are advised to read the separate enclosed instructions before beginning the writing of assignments.*

Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme/ Practical classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

ANSWER ALL TOPICS

Paper I: Algebra

1. a) State Sylow's theorem. If G is a group of order pqr, p<q<r being primes. Prove
   i. Sylow r-subgroup is normal in G
   ii. G has a normal subgroup of order qr
   iii. If $q \nmid r-1$, then Sylow q-subgroup is normal in G.

   b) Prove that any two non-zero elements a, b in a Euclidean domain R have a g.c.d. 'd' and it is possible to write d = $\lambda a + \mu b$, for some $\lambda, \mu \in R$.
2. a) If R is a U.F.D then show that any $f(x) \in R[x]$ is an irreducible element of R[x] if and Only if f is an irreducible element of R or f is an irreducible primitive polynomial of R[x].

   b) Show that the set of all real valued continuous functions y = f(x) satisfying the differential equation $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$ is a vector space over R. Find a basis of this.
3. a) Define a characteristic and a minimal polynomial for the linear operator T. If T is a linear operator on a n-dimensional space V. Show that characteristic and minimal polynomial for T have the same roots. Are characteristic and minimal polynomials for T is same? Justify.

   b) Determine the Jordon canonical form for the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and determine a matrix P which conjugates this matrix into its Jordon canonical form.

Paper II: Analysis –I

1) a) Prove that R^1 is not compact.
   b) Construct a compact set of real numbers whose limit points form a countable set.
2) a) Show that $\{1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} - \log n\}$ converges.

b) Let $f' \geq 0$ on [a, b], Show that $2f\left(\frac{a+b}{2}\right) \leq f(a)+f(b)$

3) a) Assume $f^2 \in R[a,b]$, Does $f \in R[a,b]$?
b) If f is monotonic on [a, b], Show that f is of bounded variation on [a, b].

Paper III: Analysis-II

1) a) Suppose $x_n > 0$, $s_n = x_1 + x_2 + x_3 ... + x_n$ and $\sum x_n$ diverges, then prove that $\sum \frac{x_n}{1+x_n}$ diverges.

b) If $\sum x_n$ is a series of complex numbers which converges absolutely then prove that every rearrangement of $\sum x_n$ converges and converges to the same sum.

2) Assume that $\{f_n\}$ is a sequence of monotonically increasing functions on R^1 with $0 \leq f_n(x) \leq 1$ *for all* x *and* n. Then prove that there is a function f and a sequence $\{n_k\}$ such that $f(x) = \lim_{k\to\infty} f_{n_k}(x)$ for every $x \in R^1$. Further if f is continuous, prove that $f_{n_k} \to f$ uniformly on R^1.

3) a) Suppose $f(x).f(y) = f(x+y)$ *for all real* x & y, then prove that $f(x) = e^{kx}$ where k is a constant. Assume that f is differentiable and continuous.

b) Suppose f is a differentiable mapping of R^1 into R^3, such that $|f(t)| = 1$ for every t. Prove that,
$f'(t).f(t) = 0$. Interpret this result geometrically.

Paper IV: Differential Equations

1. a) Obtain the eigen value and eigen function for the boundary value problem
$x'' + \lambda x = 0, x(0) = 0, x(0) + x'(\pi) = 0$

b) Construct the Green's function for the boundary value problem
$x'' = sint,\ x(0) + x(1) = 0,\ x'(0) + x'(1) = 0$

2. Using Laplace transform technique solve the following initial value problem
$y'' + xy' + y = 0,\ y(0)=1,\ y'(0)=0$

3. Reduce the following equation to it canonical form and hence solve it
$e^x \frac{\partial^2 u}{\partial x^2} + e^y \frac{\partial^2 u}{\partial y^2} = u.$

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