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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS (2010-11)
Course: **M.Sc. MATHEMATICS (Final)**

Note: *Students are advised to read the separate enclosed instructions before beginning the writing of assignments.*

Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

ANSWER ALL TOPICS

PAPER V: COMPLEX ANALYSIS

1. a) If f and g are functions that satisfies the C-R equations at a point. z∈ C, then prove with counter example that f+g and fg also satisfies the C-R equations at z.
   b) Find the most general linear transformation of the circle $|z| = R$ into itself.
2. State and prove Morera's theorem. Further, show that $f(z) = \int_0^\infty \frac{e^{zt}}{1+t} dt$ is analytic in the left half plane $L = \{z \in C : \text{Re } z < 0\}$ using Morera's theorem.
3. a) Suppose f is analytic in $|z| \le 1$ with $|f(z)| \le 2$ for $|z| = 1$, Im $z \ge 0$ & $|f(z)| \le 3$ for $|z| = 1$, Im $z < 0$ then prove that $|f(0)| \le \sqrt{6}$.
   b) How many roots of the equation $Z^7 - 2Z^5 + 6Z^3 - Z + 1 = 0$ lie in $|z| < 1$.

PAPER VI: TOPOLOGY

1. a) Find a topological space and a compact subset whose closure is not compact.
   b) Show that the rationals Q are not locally compact.

2. a) If A and B are path-connected subsets of a space & if $A \cap B$ is nonempty, prove that $A \cup B$ is path-connected.
   b) Find a family of union closed subsets of the real line whose union is not closed.

3. a) Prove that the product of two second countable spaces is second countable, and that the product of two separable spaces is separable.
   b) Every regular, second countable space is normal.

PAPER VII: MEASURE THEORY AND FUNCTIONAL ANALYSIS

1. a) Construct a sequence $\langle f_n \rangle$ of non-negative, Riemann integrable functions such that f_n increases monotonically to f. What does this imply about changing the order of integration & the limiting process.
   b) Construct a monotone function on $[0,1]$ which is discontinuous at each rational point.
2. a) Let f be defined by $f(0) = 0$ and $f(x) = x^2 \sin\left(\frac{1}{x^2}\right)$ for $x \neq 0$. Is f of bounded variation on $[-1, 1]$?
   b) Define a complete metric space. Show that $[0,1]$ is complete, where as $(0,1]$ is not complete. Is Q (The set of rationals) complete? Justify.
3. a) Define a totally bounded metric space. Show that the bounded interval is a totally bounded metric space.
   b) State open mapping theorem and closed graph theorem. Give atleast two examples to show that the closed graph theorem and open mapping theorem may not hold if the normed spaces X and Y are not Banach spaces.

PAPER VIII: NUMERICAL ANALYSIS

1. a) Obtain the number of roots of the polynomial $x^4 - 3x^3 + 20x^2 + 44x + 54 = 0$ in the interval $[0,4]$ using sturm sequences.
   b) Reduce the matrix $\begin{pmatrix} 1 & 1 & \frac{1}{2} \\ 1 & 1 & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & 2 \end{pmatrix}$ into tridiagonal form using House holder's method.
2. Construct a least square approximation polynomial of degree $\leq$ 1 & 2 for the function $f(x) = e^x \sin x$ in the interval $[0,1]$ with respect to the Wight function $w(x) = 1$.
3. a) Solve an IVP $y' = y + x^2$, $y(0) = 1$ using Adams Predictor method, choose h=0.2
   b) Solve the heat conduction equation $u_t = u_{xx}$ $u(0,t) = u(1,t) = 0$ and $u(x,0) = x - x^2$, choose h=0.25, k=0.025, using Crank-Nicolson method.

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