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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS (2010-11)

Course: M.Sc. MATHEMATICS (Previous)

Note: *Students are advised to read the separate enclosed instructions before beginning the writing of assignments.*

Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

ANSWER ALL TOPICS

PAPER I: ALGEBRA

1 .a) If G is a simple group of order 60, then show that $G \cong A_5$.

b) If G is a finite cyclic group of order $'n'$, then determine $Aut\ G$.

2. a) Let R be a principal ideal domain which is not a field, show that an ideal $A = (a_0)$ is a maximal ideal if and only if a_0 is an irreducible element.

b) If W is a subspace of a vector space V, define Annihilator $A(W)$ of W. If W is a subspace of R^5 spanned by $v_1 = (2, -2, 3, 4, -1)$; $v_2 = (-1, 1, 2, 5, 2)$; $v_3 = (0, 0, -1, -2, 3)$; $v_4 = (1, -1, 2, 3, 0)$ determine $A(W)$.

3. a) Let T be a linear operator on a finite dimensional vector space V. Prove that T is triangular if and only if minimal polynomial for T is of the form

$$P(x) = (x - c_1)^{r_1}\ (x - c_2)^{r_2} \ldots\ (x - c_k)^{r_k}$$

b) Find the degree of the splitting field of $x^5 - 3x^3 + x^2 - 3$ over Q.

PAPER II: ANALYSIS-I

1. a) Show that $\sqrt{3}$ is irrational.

b) Show that irrationals are uncountable.

2. a) Is (a, b) open on R^2?

b) Construct a bounded set of a real numbers with exactly three limit points.

3. a) Construct a component set of real numbers with a countable set of limit points.

b) Let E be a dense subset of a metric space X, and let f be uniformly continues real function defined on E. Prove that f has continues extension from E to X.

PAPER III: ANALYSIS-II

1. a) If $\{f_n\}$ is a sequence of continuous functions on A, and if $f_n \to f$ uniformly on A, then prove that f is continuous on A.

   b) Let $f_n(x) = \dfrac{1}{(1+n^2x^2)}$ if $0 \le x \le 1$, $n = 1,2,.....$ prove that $\{f_n\}$converges point wise but not uniformly on $[0,1]$. Is term-by-term integration permissible?

2. a) Give an example of a continuous(not monotonic) function f with the property that $\sum_{n=1}^{\infty} f(n)$ converges & $\int_0^{\infty} f(x)dx$ diverges.

   b) Show that the power series $\sum_{n=0}^{\infty} a_n x^n$ & $\sum_{n=1}^{\infty} n a_n x^{n-1}$ have the same radius of convergence.

3. a) If f is a C^1 - mapping of an open set $A \subset R^n$ into R^n and if $f^1(x)$ is invertible for every $x \in A$. Then prove that f(U) is open subset of R^n for any open set $U \subset A$.

   b) Assume $T \in L(X, Y)$ and $T_x = 0$ only when x=0. Then prove that T is 1-1.

PAPER IV: DIFFERENTIAL EQUATIONS

1. a) Obtain the solution of non homogenous equations

   i) $y''' - 8y = e^{ix}$

   ii) $y'' - 2iy' - y = e^{ix} - 2e^{-ix}$ using method of variation of parameters.

   b) Show that solution of $x^2y'' + 4xy' + (2 + x^2)y = 0$ are of the form $\phi_1 = \dfrac{\varphi_1}{x^2}$ $\phi_2 = \dfrac{\varphi_2}{x^2}$.

   Hence find all the solutions of $x^2y'' + 4xy' + (2 + x^2)y = x^2$

2. Find the solution of $x^2y'' - xy' + y = x^2$, $y(0) = 0$, $y'(0) = 1$ using Laplace Transform method.

3. a) Obtain the solution of the equation $u_t = u_{xx}$ $0 \le x < 1$ $(t > 0)$ subjected to the boundary conditions $u(0,t) = k_1$ $u(1,t) = k_2$ and initial condition $u(x,0) = f(x)$.

   b) Reduce the following equation into its canonical form

$$e^y \frac{\partial^2 u}{\partial x^2} + e^x \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = e^y \cos x.$$

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