Q.P. Code – 50622

First Year B.Sc., Degree Examinations, OCTOBER/NOVEMBER 2016

(Directorate of Distance Education)

(DSA 230) Paper I – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX** full questions of the following choosing at least **ONE** from each Part.*

PART – A

1. (a) (i) If $(a, b) = 1$, a/c and b/c then prove that ab/c.

(ii) Solve $6x + 2 \equiv 3 \pmod{10}$. **2 + 2**

(b) Find the G.C.D. of 216 and 6125 and express it in the form $ma + nb$, where $m, n \in Z$. **5**

(c) State and prove Chinese Remainder Theorem. **6**

2. (a) (i) Define Identity relation. Give an example.

(ii) Prove that the composition of functions is associative. **2 + 2**

(b) In $N \times N$, where N is the set of all natural numbers, the relation R is defined by $(a, b)\ R\ (c, d)$ iff $a + d = b + c$. Show that R is an Equivalence relation. **5**

(c) Prove that the set of all rational numbers Q is countable. **6**

PART – B

3. (a) (i) Evaluate $\lim_{x \to 0} x \sin \frac{1}{x}$.

(ii) Find the n^{th} derivative of a^{mx}, where $a \neq 0$ and m is a constant. **2 + 2**

(b) Discuss the differentiability of the function **5**

$$f(x) = \begin{cases} 1 + 2x & \text{if } -1 \le x < 0 \\ 1 - 3x & \text{if } 0 \le x < 1 \\ x - 3 & \text{if } 1 \le x \le 2 \end{cases} \quad \text{at } x = 0, 1$$

(c) If $y = e^{m \sin^{-1} x}$ then show that

$(1 - x^2) y_{n+2} - (2n + 1) x\, y_{n+1} - (n^2 + m^2) y_n = 0$. **6**

4. (a) (i) Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$.

(ii) If $x = a\cos^3 t,\ y = a\sin^3 t$ find $\frac{ds}{dt}$. **2 + 2**

(b) If $y^2 = 4a(x+a)$, then find the Pedal equation of the curve. **5**

(c) Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$ is another cycloid. **6**

PART – C

5. (a) (i) Find the parametric representation of the line through the two points (1, –1, 1) and (2, 3, 0).

(ii) Find the equation to the plane through (7, 4, 5) which is parallel to the plane $2x - 3y - 6 = 0$. **2 + 2**

(b) Find the equation of the plane passing through the point (4, –1, 0) and the line $x = t,\ y = 2t,\ z = 3t$. **5**

(c) Find the mutual position of the lines l_1 and l_2 given by

$$l_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = t$$

$$l_2 : \frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7} = s$$ **6**

6. (a) (i) Find the equation of the sphere having the points (2, 1, –3) and (1, –2, 4) as the ends of a diameter. Find its centre and radius.

(ii) Find the asymptotes parallel to co-ordinate axes for the curve $x^2y - 3x^2 - 5xy + 6y + 2 = 0$. **2 + 2**

(b) Find the position and nature of the double points of the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$. **5**

(c) Find the volume of the solid generated by the revolution of the curve astroid. $x^{2/3} + y^{2/3} = a^{2/3}$. **6**

PART – D

7. (a) (i) If *A* is symmetric matrix and *K* is any scalar. Then prove that *KA* is also symmetric matrix.

(ii) Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$. **2 + 2**

(b) Find the inverse of the matrix $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ by elementary row operations. **5**

(c) Solve completely the System of Equations

$$\begin{aligned} x + 2y + 3z &= 0 \\ 2x + 3y + 4z &= 0 \\ 7x + 13y + 19z &= 0 \end{aligned}$$ **6**

8. (a) (i) Evaluate $\int \frac{1 - \tan x}{1 + \tan x} dx$.

(ii) Evaluate $\int_{-\pi/2}^{\pi/2} \cos^8 x \, dx$. **2 + 2**

(b) Evaluate $\int \frac{x^2 + 2x + 3}{\sqrt{x^2 + 1}} dx$. **5**

(c) Prove that $\int_0^{\pi/4} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$. **6**
