Q.P. Code – 50722

Second Year B.Sc., Degree Examinations, OCTOBER/NOVEMBER 2016

(Directorate of Distance Education)

(DSB 230) Paper II – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX** full questions of the following choosing at least **ONE** from each Part.*

PART – A

1. (a) (i) Find the degree and order of the differential equation $\frac{d^3y}{dx^3} + 6y + xy = 6\left(\frac{dy}{dx}\right)^2$.

(ii) Solve $(x^2+1)\frac{dy}{dx} = 1$. **2 + 2**

(b) Solve : $\frac{dy}{dx} = \frac{6x-2y-7}{3x-y+4}$. **5**

(c) Solve : $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$. **6**

2. (a) (i) Solve : $p^2 - 5p - 6 = 0$.

(ii) Find the general and singular solution of $y = px + \sin^{-1} p$. **2 + 2**

(b) Solve : $y - 2px + yp^2 = 0$. **5**

(c) Show that the family of parabolas $y^2 = 4a(x+a)$ is self orthogonal. **6**

PART – B

3. (a) (i) Solve : $(D^2 - 7D + 12)y = 0$, where $D = \frac{d}{dx}$.

(ii) Solve : $\frac{d^2y}{dx^2} + 4y = 0$. **2 + 2**

(b) Solve : $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$. **5**

(c) Solve : $(D^2 + 3D + 2)y = e^{2x} \sin x$, where $D = \frac{d}{dx}$. **6**

4. (a) (i) Evaluate : $\lim_{x \to 0}\left[\frac{x - \sin x}{x^3}\right]$.

(ii) Evaluate : $\lim_{x \to 1}\left[\frac{2}{x^2 - 1} - \frac{1}{x - 1}\right]$. **2 + 2**

(b) Verify the Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in $[2, 4]$. **5**

(c) Expand $\log(\sec x)$ by Maclaurin's expansion upto the term containing x^6. **6**

PART – C

5. (a) (i) Show that, if every element of a group G is its own inverse, then G is abelian.

(ii) Find the number of generators of the cyclic group of order 30. **2 + 2**

(b) List all the subgroups of $(z_6, +_6)$. **5**

(c) State and prove Lagrange's theorem. **6**

6. (a) (i) Solve : $2x - 3 < 5x + 3 < 2x + 3$.

(ii) Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$. **2 + 2**

(b) Find the order of the permutation

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 & 7 & 8 & 1 & 2 \end{pmatrix}$$

and verify whether it is odd or even. **5**

(c) State and prove Fermat's theorem. **6**

PART – D

7. (a) (i) Prove that every convergent sequence is bounded.

(ii) Evaluate : $\lim_{n \to \infty}\left(\frac{n+1}{n-1}\right)^n$. **2 + 2**

(b) Prove that the sequence $\{x_n\} = \left\{\frac{3n+4}{2n+1}\right\}$ is

(i) monotonically decreasing

(ii) bounded

(iii) converges to $\frac{3}{2}$. **5**

(c) Discuss the behaviour of the sequence $\{x^{1/n}\}$, where $x > 0$. **6**

8. (a) (i) Test the convergence of the series $1^3 + 2^3 + 3^3 + \cdots + n^3 + \cdots$.

(ii) Discuss the convergence of the series :

$$\frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \cdots.$$ **2 + 2**

(b) Test the convergence of the series : **5**

$$1 + \frac{3}{7}x + \frac{3\cdot 6}{7\cdot 10}x^2 + \frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3 + \cdots, \text{ where } x > 0.$$

(c) Find the sum of the following series : **6**

$$1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \frac{1+2+2^2+2^3}{4!} + \cdots.$$
