Q.P. Code – 50824

Third Year B.Sc., Degree Examinations, OCTOBER/NOVEMBER 2016

(Directorate of Distance Education)

(DSC 231) Paper IV – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

Answer any ***SIX*** *of the following.*

PART – A

1. (a) (i) Evaluate $\int_C [(x+2y)dx + (4-2x)dy]$ around the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ in the counter clockwise.

(ii) Evaluate $\int_0^2 \int_0^{x^2} x(x^2+y^2)\,dy\,dx$. **2 + 2**

(b) Evaluate $\int_C (x^2-2xy)dx + (x^2y+3)dy$ around the boundary of the region defined by $y^2=8x$ and $x=2$. **5**

(c) Evaluate $\iint_A (x^2+y^2)\,dx\,dy$, where A is the area enclosed by the lines $y=4x$, $x+y=3$, $y=0$ and $y=2$. **6**

2. (a) (i) Evaluate $\int_0^1 \int_0^2 (x+y)\,dx$.

(ii) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\,dx\,dy\,dz$. **2 + 2**

(b) Find the surface area of the cylinder $x^2+y^2=4$ cut by the cylinder $x^2+z^2=4$. **5**

(c) Evaluate $\iiint_R xyz\,dx\,dy\,dz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$ by transforming into cylindrical polar co-ordinates. **6**

3. (a) (i) Express the integral $\int_0^1 x^m \left(1-x^n\right)^p dx$ in terms of the Beta function.

(ii) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **2 + 2**

(b) Prove that $\frac{\beta(m+1,\, n)}{m} = \frac{\beta(m,\, n+1)}{n} = \frac{\beta(m,\, n)}{m+n}$. **5**

(c) If n is a positive integer, then prove that $\Gamma\left(n+\frac{1}{2}\right) = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^n}\sqrt{\pi}$ and further find $\Gamma\left(\frac{9}{2}\right)$. **6**

4. (a) (i) Define Riemann integration.

(ii) Show that $\int_0^2 [x]dx = 1$. **2 + 2**

(b) State and prove condition for Riemann integrability. **5**

(c) If f is $R[a,\, b]$ and if there exists a primitive function ϕ of f, then prove that $\int_a^b f(x)dx = \phi(b) - \phi(a)$. **6**

PART – B

5. (a) (i) Find the Wronskin for the equation $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.

(ii) Find the part of complementary function for the equation

$xy'' - 2(x+y)y' + (x+2)y = (x-2)e^{2x};\ (x>0)$. **2 + 2**

(b) Solve by changing the dependent variable

$x^2y'' - 4x^3y' + \left(4x^4 - 2x^2 + 5\right)y = x^4 e^{x^2},\ (x>0)$

by removing the first derivative. **5**

(c) Solve : $y'' + (2\cos x + \tan x)y' + y\cos^2 x = \cos^4 x$ by changing the independent variable. **6**

6. (a) (i) Verify the condition of exactness of the equation

$$\left(2x^2+3x\right)\frac{d^2y}{dx^2}+\left(6x+3\right)\frac{dy}{dx}+2y=\left(x+1\right)e^x.$$

(ii) Solve $\frac{dx}{y^2z}=\frac{dy}{x^2z}=\frac{dz}{y^2x}$. **2 + 2**

(b) Solve $\frac{dx}{y^2+yz+z^2}=\frac{dy}{z^2+zx+x^2}=\frac{dz}{x^2+xy+y^2}$. **5**

(c) Solve $\left(x^2+1\right)y''-2xy'+2y=6\left(x^2+1\right)^2$, given $y=-2$ when $x=-2$ and $y=4$ when $x=2$ by the method of variation of parameter. **6**

7. (a) (i) Verify the condition for integrability of the equation

$$\left(y^2+yz\right)dx+\left(xz+z^2\right)dy+\left(y^2-xy\right)dz=0.$$

(ii) Construct a partial differential equation by eliminating a and b from $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$. **2 + 2**

(b) Solve : $zpy^2=x\left(y^2+z^2q^2\right)$. **5**

(c) Solve : $p\left(1+q^2\right)=q\left(z-1\right)$. Also show that there is no singular solution. **6**

8. (a) (i) Find the Fourier coefficient a_n for $f\left(x\right)=e^{-ax}$ in the interval $\left(-\pi,\pi\right)$.

(ii) Obtain the half range Fourier sine series of $f\left(x\right)=x,\ 0\le x\le L$. **2 + 2**

(b) Obtain the Fourier series of $f\left(x\right)=\begin{cases} x, & -\frac{\pi}{2}\le x\le\frac{\pi}{2} \\ \pi-x, & \frac{\pi}{2}\le x\le\frac{3\pi}{2} \end{cases}$

$f\left(x\right)$ being a periodic function with period 2π. **5**

(c) Find the Fourier series for $f\left(x\right)=\begin{cases} -1 & -1<x<0 \\ 2x & 0<x<1 \end{cases}$. **6**
