Third Year B.Sc. Degree Examinations

December 2017

(Directorate of Distance Education)

Paper – IV: DSC 231: MATHEMATICS

Time: 3 hrs] *[Max. Marks: 90*

Instructions to candidates:

*Answer any **SIX** of the following.*

PART – A

1. a) i) Evaluate $\int_C \left(x^2 - y\right) dx + \left(x + y^2\right) dy$ where C is the curve given by $x = t,\ y = t^2 + 1$ and $0 \le t \le 1$.

ii) Evaluate $\int_0^a \int_0^b \left(x^2 + y^2\right) dx\ dy$. (2 + 2)

b) Evaluate $\int_C (x + y + z)\, ds$ where C is the line joining the points $(1, 2, 3)$ and $(4, 5, 6)$ (5)

c) Evaluate $\iint_R y\, dx\, dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (6)

2. a) i) Evaluate $\int_0^3 \int_0^2 \int_0^1 (x + y + z)\, dz\, dy\, dx$

ii) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dx\, dy\, dz$. (2 + 2)

b) Find the surface area of the portion of the plane $3x + 2y + 6z = 12$ inside the cylinder $x^2 + y^2 = 1$. (5)

c) Find the volume of the cylinder bounded by $x^2 + y^2 = 4$ and the plane $y + z = 3$ and $z = 0$. (6)

3. a) i) Prove that $\beta(m, n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\, dy$.

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ii) Evaluate $\dfrac{\Gamma(3)\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{9}{2}\right)}$. (2 + 2)

b) Show that $\displaystyle\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$. (5)

c) Prove that $\dfrac{\Gamma(n)\,\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(2n)} = \dfrac{\sqrt{\pi}}{2^{2n-1}}$ (6)

4. a) i) Define upper and lower Riemann integrals

ii) If $f(x)=\begin{cases}0 & if \quad x \text{ is rational}\\ 1 & if \quad x \text{ is irrational}\end{cases}$

then prove that f is not Riemann integrable over the interval $[a, b]$. (2 + 2)

b) Prove that every monotonic function is R–integrable. (5)

c) If $f(x)$ is bounded and integrable over $[a, b]$ then prove that $|f(x)|$ is also integrable over $[a, b]$ and $\left|\displaystyle\int_a^b f(x)\,dx\right| \le \displaystyle\int_a^b |f(x)|\,dx$. (6)

PART – B

5. a) i) Find the part of complimentary function of

$$x^2\frac{d^2y}{dx^2}-\left(x^2+2x\right)\frac{dy}{dx}+(x+2)\,y = x^3e^x.$$

ii) Find the Wronskian for the equation $\dfrac{d^2y}{dx^2}+y=\sec x$ (2 + 2)

b) Solve by changing the independent variable

$$\cos x\,\frac{d^2y}{dx^2}+\sin x\frac{dy}{dx}-2\cos^3 x\,y = 2\cos^5 x$$ (5)

c) Solve $x^2\dfrac{d^2y}{dx^2}-2x(1+x)\dfrac{dy}{dx}+2(1+x)\,y = x^3$ by changing the dependent variable, given that $y=\dfrac{1}{2},\ y'=1$ when $x=1$. (6)

6. a) i) Test for exactness of the equation

$$x^2(1+x)\frac{d^2y}{dx^2}+2x(2+3x)\frac{dy}{dx}+2(1+3x)\,y=0.$$

ii) Verify the integrability of $(yz+2x)dx+(zx-2z)dy+(xy-2y)dz=0$ (2 + 2)

b) Solve $x^2\dfrac{d^2y}{dx^2}-4x\dfrac{dy}{dx}+6y=x^4\sin x$ by the method of variation of parameters. (5)

c) Solve $\dfrac{dx}{x^2+y^2+yz}=\dfrac{dy}{x^2+y^2-zx}=\dfrac{dz}{z(x+y)}$ (6)

7. a) i) Form the partial differential equation from $2z=a\sin x+b\cos y$ by eliminating constants a and b.

ii) Solve $p\tan x+q\tan y=\tan z$ (2 + 2)

b) Solve $z^2\left(p^2+q^2+1\right)=1$ (5)

c) Find the complete integral $z=px+qy+p^2+q^2$ by Charpit's method. (6)

8. a) i) Obtain the Fourier coefficient a_n for the function $f(x)=x^2$ where $-\pi<x<\pi$ and $f(x+2\pi)=f(x)$.

ii) Find the Fourier coefficient a_0 for $f(x)=\begin{cases}-1 & if \quad -1<x\le 0\\ 2x & if \quad 0<x<1\end{cases}$ and $f(x+2)=f(x)$ (2 + 2)

b) Find the Fourier series of $f(x)=\begin{cases}1 & if \quad \dfrac{-\pi}{2}<x\le\dfrac{\pi}{2}\\ -1 & if \quad \dfrac{\pi}{2}<x<\dfrac{3\pi}{2}\end{cases}$ and $f(x+2\pi)=f(x)$ (5)

c) Obtain the Fourier series expansion of $f(x)=x^2 \quad -\pi\le x\le\pi$ and $f(x+2\pi)=f(x)$ and hence deduce $\sum_{n=1}^{\infty}\dfrac{1}{n^2}=\dfrac{\pi^2}{6}$ (6)

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