DPB – 510 MAT

M.Sc. Final Examination, August/September 2008
Directorate of Correspondence Course
MATHEMATICS
Paper – V : Complex Analysis

Time : 3 Hours Max. Marks : 70

Note : Answer any five questions.
All questions carry equal marks.

1. a) Prove that $\left|\frac{a-b}{a-\bar{a}b}\right|<1$; if $|a|<1$ and $|b|<1$.

   b) Obtain the spherical representation of the extended complex plane.

   c) Show that z and z' correspond to diametrically opposite points on the Riemann sphere iff $z'z^{-1}=-1$. **(4+6+4)**

2. a) Let $f(z)=\sum_{n=0}^{\infty}a_n z^n$ $|z|<R$, where R is the radius of convergence of f. Show that f is analytic in $|z|<R$.

   b) If $\sum a_n$ converges, show that $\lim_{z\to 1}\sum a_n z^n=\sum a_n$ is in such a way that $\frac{|1-z|}{1-|z|}$ is bounded. **(8+6)**

3. a) Show that a function $f=u+iv$ is analytic in a domain D, iff the partial derivatives u_x, u_y, v_x and v_y exists, continuous and satisfy C-R conditions.

   b) Prove that Cross ratio is invariant under any bilinear transformation.

   c) Find the linear transformation which carries 0, i, – i int 1, – 1, 0. **(6+4+4)**

4. a) State and prove Cauchy theorem for an open disk D.

   b) Verify Cauchy's integral theorem for the function $f(z)=z^2$, defined in the region bounded by the triangle with vertices (0, 0), (3, 0) and (3, 1).

   c) State and prove fundamental theorem of algebra. **(5+4+5)**

P.T.O.

5. a) State and prove maximum modulus principle and use it to derive Schwartz's lemma.

   b) If f(z) is meromorphic inside a closed contour C and has no zeros on C, then prove that

$$\frac{1}{2\pi i}\int \frac{f'(z)}{f(z)}\,dz = N - P,$$

   Where N is the number of zeros and P the number of poles inside C. **(7+7)**

6. a) State and prove Rouche's theorem and use it to prove the fundamental theorem of algebra.

   b) Derive Poisson's formula for a harmonic function. **(7+7)**

7. a) State and prove Cauchy's residue theorem.

   b) Integrate any two by method of residues :

   i) $\int_0^{2\pi} \frac{d\theta}{a+\cos\theta}, a > 1$

   ii) $\int_0^{\infty} \frac{\cos x}{a^2+x^2}\,dx\ (a > 0)$

   iii) $\int_{-\infty}^{\infty} \frac{dx}{1+x^4}$ **(7+7)**

8. a) State and prove Mittag-Leffler's theorem.

   b) Show that $\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(\frac{1-z^2}{n^2} \right)$. **(7+7)**

DPB – 520 MAT

M.Sc. Final Examination, August/September 2008
Directorate of Correspondence Course
MATHEMATICS
Topology

Time : 3 Hours

Max. Marks : 70

*Note : 1) Answer any **FIVE** questions.*
*2) All questions carry **equal** marks.*

1. a) State and prove Kuratowski closure operator theorem. **10**

   b) Let X be a space, $A, B \subset X$. Show that $\overline{A \times B} = \overline{A} \times \overline{B}$. **4**

2. a) Let X and Y be spaces. Show that $f : X \to Y$ is a homeomorphism iff $\overline{f(A)} = f(\overline{A})$, $\forall A \subset X$. **10**

   b) Show that a second countable space is separable. **4**

3. a) If X and Y are compact, show that $X \times Y$ is compact. **8**

   b) Show that a compact metric space is second countable. **6**

4. a) Show that a connected metric space with more than one point is uncountable. **8**

   b) Show that a connected space may not be path connected. **6**

5. State and prove the Urysohn lemma. **14**

6. Show that a regular, Lindelöf space is

   i) Normal. **7**

   ii) Paracompact. **7**

7. Show that :

   i) A locally compact Hausdorff space is completely regular. **6**

   ii) If X and Y are normal, $X \times Y$ may not be normal. **8**

8. a) Show that a paracompact Hausdorff space is regular. **4**

   b) Show that a metric space is paracompact. **10**

DPB – 530 Mat

Final Year M.Sc. (Mathematics) Degree Examination, August/September 2008

Directorate of Correspondence Course

Paper – VII : MEASURE THEORY AND FUNCTIONAL ANALYSIS

Time : 3 Hours | Max. Marks : 70

*Note : 1) Answer **any FIVE** questions.*

*2) **All** questions carry **equal** marks.*

1. a) Define the Lebesgue outer measure of a set A. Prove that the Lebesgue outer measure of an open interval is its length.

   b) Define a measurable set. If A and B are measurable sets, then prove that $A \cup B$ and $A \cap B$ are also measurable sets.

   c) Construct a non measurable set. **(6+4+4)**

2. a) Let $\{E_i\}$ be a sequence of Lebesgue measurable sets, then prove that

   $m\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m(E_i)$

   b) Define measurable function on a measurable set. If f is a measurable function and B is a Borel set then prove that $f^{-1}(B)$ is a measurable set.

   c) Let f and g be two functions defined on a measurable set A. If f = g almost every where and f is measurable then prove that g is also measurable. **(6+4+4)**

3. a) State and prove Bounded convergence theorem.

   b) State and prove Monotone convergence theorem.

   c) Let $\{u_n\}_{n=1}^{\infty}$ be a sequence of non-negative measurable functions and let $f = \sum_{n=1}^{\infty} u_n$ almost everywhere on a measurable set E, then prove that $\int_E f = \sum_{n=1}^{\infty} \int_E u_n$ **(6+4+4)**

P.T.O.

4. a) Let g be integrable over a measurable set E and $\{f_n\}$ a sequence of measurable functions such that $\{f_n\} < g$ on E and for almost all x in E with $f(x) = \lim_{n\to\infty} f_n(x)$ then prove that $\int_E f = \lim \int_E f_n$.

   b) State and prove Vitali's covering lemma.

   c) Prove that the normed linear space l_p^n is complete for $1 \le p < \infty$. **(5+5+4)**

5. a) Let S be the set of continuous functions on a closed interval [a, b], let $d : S \times S \to R$ be defined by $d(x, y) = \int_a^b |x(t) - y(t)dt$. Is S complete ? Justify.

   b) Define a separable space. Show that l_p^n is separable, where $1 \le p < \infty$. **(7+7)**

6. a) Show that a normed linear space is Banach space if and only if $S' = \{x : \| x \| = 1\}$ is complete.

   b) Define a continuous linear transformation. If N and N′ are normed linear spaces over the same field and $T : N \to N'$ is a linear transformation then prove that the following are equivalent :

      i) T is continuous.

      ii) T is continuous at origin.

      iii) there exists $k > 0$ such that $\forall x \in N, \| T(x) \| \le k \| x \|$.

      iv) T (s) is bounded in N′ where $S = \{x \in N : \| x \| \le 1\}$ is the closed unit ball in N. **(7+7)**

7. a) State and prove Hann-Banach theorem for reals.

   b) State open mapping theorem. Hence state and prove closed graph theorem. **(7+7)**

8. a) State and prove Uniform boundedness theorem.

   b) Prove that a non-empty subset X of a normed linear space N is bounded if and only if f (X) is bounded for each $f \in N^*$.

   c) If T is an operator on a normed linear space N, then prove that its conjugate T^* is an operator on N^* and the $T \to T^*$ is an isometric isomorphism of B (N) into B (N^*) which reverses products and preserves the identity transformation. **(6+4+4)**

M.Sc. Final Year (Mathematics) Degree Examination, August/September 2008
Directorate of Correspondence Course
Paper – III : NUMERICAL ANALYSIS

Time : 3 Hours | Max. Marks : 70

Note : *1) Answer any* ***FIVE*** *questions.*
2) ***All*** *questions carry* ***Equal*** *marks.*

1. a) State the Newton-Raphson scheme to obtain the roots of an equation $f(x) = 0$.

   Apply Newton-Raphson method to find an approximate root, correct to three decimal places of the equation. $x^3 - 3x + 5 = 0$ which lies near $x = 2$. **8**

   b) Show that the order of convergence of the secant method is 1.618 approximately. **6**

2. a) Perform two steps of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$. **7**

   b) Determine the convergence factor for the Jacobi and Gauss Seidel methods for the system.

   $4x_1 + 2x_3 = 4$

   $5x_2 + 2x_3 = -3$

   $5x_1 + 4x_2 + 12x_3 = 2.$ **7**

3. a) Find all the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ by Jacobi method. **7**

   b) Determine the Hermite polynomial of degree 5 which fits the following data and hence find an approximate value of log 2.7.

x	2	2.5	3
$y = \log x$	0.69315	0.91629	1.09861
y'	0.5	0.4	0.3333

7

P.T.O.

4. a) Derive cubic spline interpolation polynomial over a grid $G = \{ a = x_0 < x_1 < ..... < x_n = b \}$. 8

b) Fit a natural cubic spline to the following data.

x	1	2	3
y	−8	−1	18

with M(1) = 0, M(3) = 0 and compute y (1.5). 6

5. a) Derive Gauss-Laguerre two-point formula. Evaluate $\int_0^\infty \frac{e^{-x}}{1+x^2} dx$ using the Gauss-Laguerre two point formula. 9

b) Evaluate $I = \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x^2+x+1} dx$ using the Gauss-Chebyshev two point and three point formulas. 5

6. a) Derive Runge-Kutta fourth order formula to solve an initial value problem. 8

b) Given $\frac{dy}{dx} = 1 + y^2$, y(0) = 0. Find y (0.8) using Predictor – Corrector formulae. 6

7. a) Describe finite difference method to solve boundary value problem. 7

b) Solve the boundary value problem $y''(x) = y(x)$, y(0) = 0 and y(1) = 1.1752 by shooting method, taking $m_0 = 0.8$, $m_1 = 0.9$. 7

8. a) Apply Crank-Nicholson method to find the solution of the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions u (x, 0) = 0, u (0, t) = 0 and u (1, t) = t. Choose $h = \frac{1}{2}$ and $k = \frac{1}{8}$. 7

b) Solve the equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ subject to the following conditions.

u (0, t) = 0, u (1, t) = 0, t > 0 and $\frac{\partial u}{\partial t}(x, 0) = 0$, u (x, 0) = $\sin^3 \pi x$, $0 \le x \le 1$. 7
