DSC – 240 (MAT-UG)

III Year B.Sc. Degree Examination, November 2008
MATHEMATICS (Paper IV)
Directorate of Distance Education Course

Time : 3 Hours | Max. Marks : 90

Note: Answer any SIX of the following:

PART–A

1. a) i) Evaluate $\int_c (x^2-y)\,dx+(y^2+x)dy$, where 'c' is the curve given by $x = t$, $y = t^2+1,\ 0 \le t \le 1$ **(2+2)**

   ii) Evaluate $\iint xye^x\, dx.dy,\ 0 < x < 1,\ 2 < y < 3.$

   b) If 'c' be a curve leading from the origin to (1,1,1), compute $\int_C 2xy\,dx+(x^2+2yz)dy+(y^2+1)dz$ **5**

   c) Evaluate $\iint_D \frac{x}{\sqrt{x^2+y^2}}\,dx.dy$. where 'D' be the part of the unit circle in the first quadrant. **6**

2. a) i) Evaluate $\int_1^2\int_0^x \frac{dx - dy}{x^2+a^2}$ **2**

   ii) Evaluate $\iiint_I xy^2z^3\ dx.dy.dz$, where I is given by $0< x < 1,\ 0 < y < 1,\ 0 < z < 1$ **2**

   b) Find the area of the surface

   $$Z = \sqrt{x^2+y^2},\ \frac{1}{16} < x^2 + y^2 < \frac{1}{4}$$ **5**

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c) Find the volume of the sphere
$x^2 + y^2 + z^2 = a^2$ using triple integration. **6**

3. a) i) Define Gamma and Beta functions : **2**

ii) Evaluate $\int_0^{\pi/2} \sin^5\theta . \cos^7\theta \, d\theta$- **2**

b) Show that $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$ **5**

c) Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} .dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$ **6**

4. a) i) Define norm of a partition and refinement of a partition. **(2+2)**

ii) Show that $f(x) = \begin{cases} 0 & \text{when 'x' is rational} \\ 1 & \text{when 'x' is irrational} \end{cases}$

in [0,1] is not R-integrable.

b) Prove that every monotonic function is integrable. **5**

c) Evaluate $\int_a^b \frac{dx}{\sqrt{x}}$ where $b > a > 0$ using Riemann sum. **6**

PART – B

5. a) i) Find the complementary function of **(2+2)**

$(1-x)y'' + xy' - y = 0, x \neq 1$

ii) Find the Wronskian of $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

b) Solve: $\frac{d^2y}{dx^2} + (2\cos x + \tan x)\frac{dy}{dx} + y.\cos^2 x = \cos^4 x$ **5**

c) Solve by changing the dependent variable

$$\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} - (a^2+1)\, y = e^x \cdot \sec x$$ by reducing it to normal form. **6**

6. a) i) Write the general form of the linear differential equation of second order with variable coefficients. **2**

ii) Write the complementary functions for the cases $1 + P + Q = 0$ and $P + Qx = 0$ **2**

b) Solve $y'' + y' \cot x - y \operatorname{cosec}^2 x = 0$, given that $\cot x$ is a solution. **5**

c) Solve $(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = (1-x)^2,\ x \neq 1$ by the method of variation of parameters. **6**

7. a) i) Examine the exactness of **(2+2)**

$$yz.dx - 2xzdy + (xy - zy^3)dz = 0$$

ii) Form a partial differential equation by eliminating 'a' and 'b' from $(x-a)^2 + (y-b)^2 + z^2 = R^2$

b) Solve $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$ **5**

c) Solve $(x^2 - y^2 - z^2)\, p + 2xyq = 2xz.$ **6**

8. a) i) Define periodic function. **(2+2)**

ii) If $f(x) = \begin{cases} -1 & -1 < x < 0 \\ 2x & 0 < x < 1 \end{cases}$ find "a_0"

b) Expand $f(x) = \begin{cases} x, & \text{for } 0 \le x < \pi \\ 2\pi - x, & \text{for } \pi \le x \le 2\pi \end{cases}$ as a fourier series. **5**

c) Find the half range cosine series for $f(x) = \pi - x$ in $(0, \pi)$. **6**