DSC – 250 (MAT-UG)

III Year B.Sc. Degree Examination, November 2008
MATHEMATICS (Paper – V)
Directorate of Distance Education Course

Time : 3 Hours | Max. Marks : 90

Note : Answer any SIX of the following.

PART – A

1. a) i) Prove that $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$ where z_1 and z_2 are complex numbers. **2**

   ii) Find the derivative of $f(z) = \frac{z-1}{z+1}$ at $2 - i$. **2**

   b) Find the equation of the circle passing through the points $1 - i$, $2i$ and $1 + i$. **5**

   c) Show that $f(z) = \cosh z$ is analytic and $f'(z) = \sinh z$. **6**

2. a) i) Show that $u = e^x \cos y + xy$ is Harmonic. **2**

   ii) Evaluate $\int_0^{1+i} (x^2 + iy)\,dz$ along the line $y = x$. **2**

   b) Find the analytic function whose real part is $e^x[x\cos y - y\sin y]$ and hence find the imaginary part. **5**

   c) Evaluate $\int_c (y - x - 3ix^2)\,dz$ along the curve C :

   i) C is the line segment from $z = 0$ to $z = 1 + i$

   ii) C consists of two line segments one from $z = 0$ to $z = i$ and the other from $z = i$ to $z = 1 + i$. **6**

3. a) i) Evalaute $\int_c \frac{e^z}{z^2} dz$ where $c : |z| = 1$ **2**

ii) Find the fixed points of the transformation $w = \frac{1-z}{1+z}$. **2**

b) State and prove Cauchy's Integral Formula. **5**

c) Find the bilinear transformation which maps $-1, 1, \infty$ onto $-i, -1, i$. **6**

4. a) i) Evaluate $\Delta^n (e^{2x+3})$ by finite differences. **2**

ii) Construct the backward difference table by using Newton's Backward interpolation formula by the given data : **2**

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

b) Find the interpolating polynomial f(x) satisfying f(0) = 0, f(2) = 4, f(4) = 56, f(6) = 204, f(8) = 496, f(10) = 980 and hence find f(3), f(5). **5**

c) For the following data find $f'(1)$ and $f''(3)$. Verify your answer by finding an interpolating polynomial : **6**

x	0	2	4	6	8
f(x)	7	13	43	145	367

PART – B

5. a) i) Find L {cosh4t sin3t}. **2**

ii) If $L\{F(t)\} = f(s)$ then prove that $L\{t\,F(t)\} = -f'(s)$. **2**

b) Find the Laplace transform of Periodic function where **5**

$$F(t) = \begin{cases} 1 & 0<t<T \\ -1 & T<t<2T \end{cases}$$

c) Express the function in terms of unit step function and hence find the Laplace transform of **6**

$$F(t)=\begin{cases} t^2 & 0<t<2 \\ t-1 & 2<t<3 \\ 7 & t>3 \end{cases}$$

6. a) i) Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$. **2**

ii) Find the Laplace transform of the convolution integral $L\left\{\int_0^t (t-\beta)\sin 3\beta \, d\beta\right\}$. **2**

b) Find $L^{-1}\left\{\text{Log}\left[\frac{s^2+4}{s(s+4)(s-4)}\right]\right\}$ **5**

c) Solve the initial value problem $x''(t)+2x'(t)+x(t)=t$ given that $x(0) = -3$, $x'(0)=3$. **6**

7. a) i) Evaluate $\int_1^7 f(x)\,dx$ using the trapezoidal rule given **2**

x	1	2	3	4	5	6	7
f(x)	2.105	2.808	3.614	4.604	5.857	7.451	9.467

ii) Show that a real root of the equation $x^3 - 4x - 9 = 0$ lies between 2 and 3. **2**

b) Use Newton - Raphson method to find a real root of the equation $x^3 + 5x - 11 = 0$ carry out 3 iterations. **5**

c) Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. Hence deduce the value of $\log_e 2$. **6**

8. a) i) Evaluate $\int_0^6 y\,dx$ from the following data by Weedle's Rule : **2**

x	0	1	2	3	4	5	6
y	1	0.5	0.2	0.1	0.0588	0.0385	0.027

ii) Use Euler's method, solve the differential equation $\frac{dy}{dx} = x+y$, $y(0) = 1$.

2 $h = 0.2$ find $y(0.2)$.

b) Using Picard's method of successive approximation find the solution of tl

e equation $\frac{dy}{dx} = 1+xy$ subject to the condition $y= 0$ when $x= 0$, upon third

5 approximation and obtain y when $x= 0.2$.

c) Use fourth order Runge-Kutta method to solve $\frac{dy}{dx} = \frac{y-x}{y-x}$, $y(0)=1$ and

6 $h=0.2$.