M.Sc. (Final) Degree Examination August / September 2009

Directorate of Correspondence Course

Mathematics

Paper : PM-10.05 : Complex Analysis

(Freshers)

Time : 3 Hours — Max. Marks : 80

Note:

1) **Answer any FIVE questions.**
2) **All questions carry EQUAL marks.**

1. a) If sum and product of two complex numbers are both real, show that the numbers are either real or one is conjugate of the other.

   b) If $|a_i| < 1$, $\lambda_i \geq 0$ for $i = 1, 2, \ldots n$ and $\lambda_1 + \lambda_2 + \ldots + \lambda_n = 1$, show that $|\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n| < 1$.

   c) State and prove necessary and sufficient condition for f(z) to be analytic.

   (3+3+10)

2. a) Find C-R equations in polar form.

   b) Find the most general harmonic polynomial of the form $ax^3 + bx^2y + cxy^2 + dy^3$ and then determine its harmonic conjugate and analytic function.

   c) Show that inside the circle of convergence, $\sum_{n=0}^{\infty} a_n z^n$ represents an analytic function f(z) which is infinitely differentiable.

   (4+4+8)

3. a) State and prove Abel's limit theorem.

   b) Show that when a circle is transformed into a circle under the map $W = 1/z$, the centre of the original circle is never mapped onto the centre of the image circle.

   c) Show that cross ratio is preserved under bilinear transformation.

4. a) Find the bilinear transformation which maps the points $z = 1, i, -i$ respectively onto the points $W = i, 0, -i$. Find the image of the region $|z| \leq 1$ from the transformation.

   b) State and prove Cauchy's theorem for a rectangle.

   c) State and prove Cauchy's inequality.

   (4+7+5)

5. a) Prove that the function which is analytic in the whole plane and satisfy the inequality $| f(z) | < | z |$, for some 'm' and all sufficiently large $| z |$ reduces to a polynomial.

   b) State and prove Taylor's Theorem.

   c) State and prove Schwarz Lemma. **(5+6+5)**

6. a) State and prove the argument principle. Give the interpretation of the name argument.

   b) Let f(z) be analytic in $0 < | z - a | < \delta$ and has a Laurent's series expansion (as applicable to the annulus $r < | z - a | < R$, with $r = 0$ and $R = \delta$),

   $f(z) = \sum_{-\infty}^{\infty} a_k (z - a)^k.$

   Then show that

   (i) f(z) has a removable singularity at z = a if and only if $a_k = 0, k < 0$ i.e., if and only if its singular part is zero.

   (ii) f(z) has a pole at z = a of order m if and only if $a_{-m} \neq 0$ and $a_k = 0$ for $k < -m$.

   (iii) f(z) has a essential singularity at z = a if and only if $a_m \neq 0$ for infinitely many negative integers m. **(6+10)**

7. a) State Rouche's Theorem and use it to show that the equation $e^z = az^n$ (if $a > e$) has n roots inside the circle $| z | = l$.

   b) Using residue theorem, solve any two of the following:

   (i) $\int_0^{2\pi} \frac{d\theta}{1 - 2p\cos\theta + p^2}$ $(0 < P < 1)$

   (ii) $\int_{-\infty}^{\infty} \frac{x^4}{x^6 + a^6} dx$ $(a > 0)$

   (iii) $\int_0^{\infty} \frac{\cos ax}{(x^2 + b^2)^2} dx$ $(a > 0, b > 0)$

(6+10)

8. a) Derive Poison's Integral formula.

   b) Show that $\pi \cot \pi z = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2z}{z^2 - n^2}$ **(10+6)**

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M.Sc. (Final) Degree Examination August / September 2009

Directorate of Correspondence Course

Mathematics

Paper : PM-10.06 : Topology

(Freshers)

Time : 3 Hours   Max. Marks : 80

Note:

1) **Answer any FIVE questions.**
2) **All questions carry EQUAL marks.**

1. a) State and prove Kuratowski closure operator theorem. **10**

   b) Let X be a space, A, B $\subset$ X. Prove i $(A \cap B)$ = i (A) $\cap$ i (B) **06**

2. a) Let X and Y be spaces and $f : X \to Y$, show $f : X \to Y$ is a homeomorphism iff $f(\overline{A}) = \overline{f(A)}$, $A \subset X$ **10**

   b) Are R^1 and R_2 homeomorphic ? Prove or disapprove. **06**

3. a) Prove a connected, a locally pathconnected space is pathconnected. **08**

   b) Let X he a space A $\subset$ X.

      (i) If $\overline{A}$ is connected, is A connected ? **04**

      (ii) If A is pathconnected is $\overline{A}$ pathconnected? **04**

4. a) If X and Y are compact, show X x Y is compact. **10**

   b) Show a compact metric space is separable. **06**

5. Show that a regular, Lindeloff space is

   (i) Normal **08**

   (ii) Paracompact **08**

6. State and prove the Urysohn lemma. **16**

7. Show that (1) a regular space need not be normal. **08**

   (2) a product of two normal spaces need not be normal **08**

8. a) Show that a metric space is para compact. **10**

   b) Is R_l (i) Paracompact ? **03**

      (ii) Metrisable? **03**

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M.Sc. (Final) Degree Examination August / September 2009

Directorate of Correspondence Course

Mathematics

Paper - PM - 10.07 : Real and Functional Analysis (Freshers)

Time : 3 Hours

Max. Marks : 80

Note:

1) **Answer any FIVE questions.**
2) **All questions carry EQUAL marks.**

1. a) Define Lebesgue outermeasure $m^*(A)$ of a subset A of R. If I is an interval in R, then prove that $m^*(I)$ is equal to the length of I.

   b) Prove that m^* is translation invariant.

   c) If $\{J_x\}$ is a finite covering of open intervals of $Q \cap [0, 1]$, show that $\Sigma\, l\ (I_n) \geq 1$. Is this true if $\{J_x\}$ is infinite.

2. a) Construct a non-measurable set.

   b) If f is non-negative and measurable, show that there exists a sequence $[\phi_n]$ of simple functions such that $\phi_n \uparrow f$.

   c) If f is a measurable function and f = g. a. e., then show that g is measurable.

3. a) State and prove Fatou's lemma.

   b) If $f(x) = \begin{cases} x \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

      Calculate the Dini derivatives of f at 0.

   c) If f is absolutely continuous on [a, b] and $f^1 = 0$ a.e., on [a, b], show that f is constant.

4. a) State and prove Vitalli covering lemma.

   b) State and prove Reisz representation theorem for $L^p[0, 1]$, $1 \leq p < \infty$.

5. a) Define a separable space. Show that lp is separable for $1 \leq p < \infty$.

   b) State and prove completion theorem.

   c) State and prove Baire's category theorem.

6. a) Define a totally bounded metric space. Prove that the metric space X is totally bounded if and only if every sequence in X has a Cauchy subsequence.

   b) State and prove Arzela Ascoli Theorem.

   c) Prove that every compact metric space is separable.

7. a) Let Y be a subspace of a Banach space X, prove that Y is closed if and only if Y is complete.

   b) Let M be a closed linear subspace of a normed linear space X. If norm of a coset x + M in the quotient space X / M is defined as $\| x + M \| = \inf \{\| x + m \| : m \in M\}$, then show that X /M is a normed linear space. Further if X is a Banach space, then so is X/M.

8. a) State and prove Hahn Banach Theorem.

   b) State and prove Closed Grap Theorem.

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M.Sc. (Final) Degree Examination August / September 2009

Directorate of Correspondence Course

Mathematics

Paper : PM - 10.08 : Numerical Analysis

(Freshers)

Time : 3 Hours Max. Marks : 80

Note:

1) **Answer any FIVE questions.**
2) **All questions carry EQUAL marks.**

1. a) Derive Newton Raphson method to find the real roots of $f(x) = 0$.

   b) Use secant method to find the real roots of $2x^3 + 3x - 5 = 0$, perform four iterations.

   c) Describe Bairstow's method to extract a quadratic factor of the form $x^2 + px + q$ from a polynomial of degree n. **(3+4+9)**

2. a) Describe Jacobi & Gauss Seidel methods to find the solution of the system $Ax = b$.

   b) Prove that the iterative scheme $x^{(k+1)} = Hx^{(k)} + c$ converges to the solution of the system $Ax = B$ when $\| H \| < 1$, were H is the iteration matrix & c is a column vector. Write H & c for Jacobi & Gauss Seidel methods.

   c) Find the solution of
   $$83x + 11y - 4z = 95$$
   $$7x + 52y + 13z = 104$$
   $$3x + 3y + 29z = 71$$
   by performing four iterations using any one of iteration matrix. **(6+5+7)**

3. a) Explain the Jacobi method to find the eigen value of eigen vector of a given real symmetric matrix.

   b) Find all the eigen values of the following matrix using Given's method
   $$A = \begin{pmatrix} 1 & 6 & 0 \\ 6 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$ **(8+8)**

4. a) Derive Lagrange's interpolation formula for the given data points (x_i, y_i), $i = 1$ 2, ..., n.

b) Discuss the convergence criteria of Hamite interpolation polynomial of degree $\leq 2n+1$.

c) Obtain the value of y when x = 2 by using Gregory Newton's forward difference interpolation for the following data.

X	1	3	4	6
Y	-3	9	30	132

(5+6+5)

5. a) Explain cubic spline interpolation over a grid $G = \{a \leq x_0 < x_1 < .... < x_n \leq b\}$

b) Determine the cubic spline S(x) for the interval [2, 3] for the following tabulated values of x & y.

X	1	2	3	4	5
Y	10	17	36	73	134

(8+8)

6. a) Obtain the least square approximation polynomial of degree ≤ 2 for $f(x) = \sqrt{x}$ on [0, 1].

b) Evaluate $\int_0^1 \frac{\sin x}{1+x} dx$ for four, six & eight equal parts using Simpson's ⅓rd rule. **(8+8)**

7. a) Derive Range Kutta IInd order method to find the solution of on initial value problem $y^1 = f(x, y)$, $y(x_0) = y_0$.

b) Solve the boundary value problem $y'' - 64y + 10 = 0$, $y(0) = y(1) = 0$ using finite difference method, choose h = 0.25 **(8+8)**

8. a) Solve the Laplace equation $U_{xx} + U_{yy} = 0$ by employing five point formulae which satisfies the following boundary conditions

$u(0, y) = 0 \quad u(x, 0) = 0$

$u(x, 1) = 100x$, $u(1, y) = 100y$ choose h = k = 1

b) Employ Crank - Nicolson method to find the following initial boundary value problem $\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta t}$, $0 \leq x \leq 1$, given $u(x, 0) = \cos 2x$, $u(0, t) = 0$, $u(1, t) = 0$ for $t > 0$. **(8+8)**

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