

Third Year B.Sc. Degree Examination
Aug/Sept 2009
Directorate of Distance Education Course

MATHEMATICS (PAPER - III)

Time : 3 Hours

Max. Marks : 90

Note : Answer any SIX full questions of the following.

PART - A

1. a) i) If H is a normal subgroup of finite order then prove that

$$O\left(\frac{G}{H}\right) = \frac{O(G)}{O(H)}$$
2
 ii) Find whether the mapping $f : (z, +) \rightarrow (2z, +)$ defined by $f(x)=2x, \forall x \in z$ is a homomorphism or not. 2
- b) Prove that a subgroup H of a group G is a normal subgroup of G iff $gHg^{-1}=H, \forall g \in G.$ 5
- c) If f is a homomorphism of a group G into a group G' with Kernel K, then prove that f(G) is isomorphic to factor group G/K. 6
2. a) i) In a ring R, prove that $a(b-c) = ab - ac \forall a, b, c \in R.$ 2
 ii) Show that the set $S = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \mid a, b, c \in \varphi \right\}$ is a subring of the ring $M_2(Z)$ of all 2x2 matrices. 2
- b) Prove that a ring is without zero divisors iff the cancellation laws hold in it. 5
- c) If $f : R \rightarrow R'$ is an isomorphism of the ring R onto R' then prove that
 - i) isomorphic image of a commutative ring is commutative
 - ii) isomorphic image of a ring with unity is a ring with unity.6
3. a) i) Prove that $1 - \sqrt{5}$ is not a unit in $Z(\sqrt{5})$ 2
 ii) Find the polynomials q(x) & r(x) if $f(x) = x^5 - x^3 + 3x - 5$ & $g(x) = x^2 + 7$ 2
- b) If a polynomial $f(x) \in F[x]$, where F is a field, is divided by $x-a$, then prove that the remainder is f(a). 5
- c) Test the equation $5x^3 + 8x^2 + 6x - 4 = 0$ for rational root. 6
4. a) i) Prove that the intersection of two normal subgroups of a group G is a normal subgroup of G. 2
 ii) Prove that the Kernel homomorphism $f : R \rightarrow R'$ of rings is a subring of R. 2
- b) Prove that a subgroup H of a group G is a normal a subgroup of G iff every right coset of H in G is a left coset of H in G. 5
- c) If u & v are ideals of a ring R. Let uv be the set of all elements that can be written as finite sums of elements of the form uv where $u \in u$ & $v \in v$ show that uv is an ideal of R. 6

PART - B

5. a) i) In any vectorspace prove that $c(-\alpha) = -c(\alpha)$ 2
 ii) Find the subspace spanned by the set $S = \{(3,0,0), (0,0,-2)\}$ in $V_3(R)$. 2
- b) If S & T are two subspaces of a vectorspace $V(F)$ then prove that their linear sum $S + T = \{\alpha + \beta \mid \alpha \in S, \beta \in T\}$ is also a subspace of V . 5
- c) Let $S = \{(1,2,1), (1,1,-1), (4,5,-2)\}$ is a subset of $V_3(R)$. Show that the vector $(2,-1,-8)$ is in $L[S]$. 6
6. a) i) Prove by an example that the union of two subspace need not be a subspace. 2
 ii) Do the vectors $(1,2,1)$ & $(2,1,1)$ form a basis for $V_3(R)$? Why? 2
- b) Prove that any two basis of a finite dimensional vectorspace V have the same finite number of elements. 5
- c) Find the basis and dimension of the subspace spanned by $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ in $V_3(R)$. 6
7. a) i) Prove that $T : V_2(R) \rightarrow V_3(R)$ defined by $T(x,y) = (x,0,y)$ is a linear transformation. 2
 ii) Prove that the range space of a linear transformation $T:V \rightarrow W$ is a subspace of W . 2
- b) Find the matrix of the linear transformation $T : V_2(R) \rightarrow V_3(R)$ defined by $T(-1,1) = (-1,0,2)$ & $T(2,1) = (1,2,1)$. 5
- c) State and prove rank-nullity theorem. 6
8. a) i) Find u_x and u_y for the function $u = \log \frac{x^2+y^2}{xy}$ 2
 ii) If $u = \frac{x^2+y^2}{\sqrt{x+y}}$ show that $xu_x + yu_y = \frac{3u}{2}$ 2
- b) If $u = f(x,y)$ is a homogeneous function of degree n in x & y then prove that $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = n(n-1)u$ 5
- c) Show that a rectangular solid of maximum volumes which can be inscribed in a sphere is a cube. 6

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