



M.Sc. Mathematics (Final) Examination, August/September 2010
Directorate of Distance Education
(Freshers)

MATHEMATICS
Paper : PM-10.05 : Complex Analysis

Time : 3 Hours

Max. Marks : 80

Note : 1) Answer any FIVE questions.

2) All questions carry equal marks.

1. a) Explain stereographic projection. 6
- b) State and prove sufficient condition for the function $f(z)$ to be analytic. 5
- c) Show that
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left[\log \left(1 + |f(z)|^2 \right) \right] = \frac{4 |f'(z)|^2}{1 + |f(z)|^2}.$$
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2. a) Show that the power series and its derived series has the same radius of convergence. 6
- b) State and prove Abel's limit theorem. 5
- c) Show that the map $\omega = 1/z$ transforms circles and lines into circles and lines. 5
3. a) Prove that the product transformations is again bilinear. 5
- b) Find the bilinear transformation which maps $\{\infty, i, 0\}$ onto $\{0, i, \infty\}$. 5
- c) Prove that the line integral $\int_{\gamma} P \cdot dx + Q \cdot dy$ defined in Ω depends only on the end points of the curve γ if and only if there is a function $U(x, y)$ in Ω with
$$\frac{\partial U}{\partial x} = P, \frac{\partial U}{\partial y} = Q.$$
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