Directorate of Correspondence Course

Third Year B.Sc. Degree Examinations

August /September 2010

(New Scheme)

MATHEMATICS

Paper - III

Time: 3 hrs.] [Max.Marks : 90

Note: *Answer any SIX full questions of the following.*

PART - A

A. *Answer the following.* **5×1=5 Marks**

1. a) i) Prove that the centre $Z = \{a \in G \mid ax = xa \forall x \in G\}$ of a group G is a normal subgroup of G.

   ii) Verify for homomorphism the mapping $f : (Z, +) \longrightarrow (2Z, +)$ defined by $f(x) = 2x \forall x \in Z$

   b) Prove that a subgroup H of a group G is normal if and only if $gHg^{-1} = H$. **5 Marks**

   c) State and prove fundamental theorem of Homomorphism of groups. **6 Marks**

2. a) i) Define an Integral Domain and give an example. **2 Marks**

   ii) Prove that Intersection of two ideals of a ring R is again an ideal. **2 Marks**

   b) Prove that every finite integral domain is a field. **5 Marks**

   c) Define Kernel of homomorphism $fiR \longrightarrow R'$ and prove that Kernel is an ideal of R. **6 Marks**

3. a) i) Prove that $2 - \sqrt{5}$ is a unit but not $1 - \sqrt{5}$ in $Z\sqrt{5}$. **2 Marks**

   ii) Find $q(x)$ and $r(x)$ if $f(x) = x^4 + 3x^3 + x^2 + 6x + 10$ and $g(x) = x^2 + x + 1$. **2 Marks**

   b) Show that $f(x) = x^4 + 4$ has four roots in Z_5 and express $f(x)$ as product of linear factors. **5 Marks**

   c) Prove that any two polynomials $f(x)$ and $g(x)$ over F have a greatest common divisor $d(x)$. Which can be expressed as $d(x) = a(x)\, f(x) + b(x)\, g(x)$ for some $a(x),\ b(x) \in F(x)$. **6 Marks**

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4. a) i) Show by example union of two normal sub groups need not be a normal subgroup. 2 Marks

ii) Find all units of Z_8 integers modulo 8. 2 Marks

b) Show that $S = \left\{ \begin{pmatrix} a & 0 \\ b & o \end{pmatrix} ; a, b \in z \right\}$ is a left ideal of $M_2(z)$ but not a right ideal. 5 Marks

c) Prove that a factor group G/N of a cyclic group G is also a cyclic group. 6 Marks

PART - B

5. a) i) Prove that in any vector space V over F, $C\alpha = 0 \Rightarrow C = 0$ or $\alpha = 0$ when $C \in F$ and $\alpha \in V$. 2 Marks

ii) Show that the set $S = \{\alpha = (x_1, x_2, ...x_n) \mid 7x_1 - x_2 = 0\}$ is a subspace of $V_n(Q)$ where $x_i \in Q \; \forall \; i = 1, 2, ...n$ 2 Marks

b) Prove that the non zero the vectors $\alpha_1, \alpha_2, ...\alpha_k$ in a vector space V are Linearly dependent if and only if some one of the vectors α_k is a linear combination of the preceding one's. 5 Marks

c) Verify the set $S = \{(1, 2, 3), (3, 1, 0), (-2, 1, 3\}$ for a basis of $V_3(R)$. In case S is not a basis. Find a basis and dimension of the subspace spanned by S. 6 Marks

6. a) i) If $T : V \longrightarrow W$ is a linear transformation of a vector space V to a vector space W then prove that $T(\alpha - \beta) = T(\alpha) - T(\beta), \forall \alpha, \beta \in V$ 2 Marks

ii) If $T : V_1(R) \longrightarrow V_3(R)$ is defined by $T(x) = (x, x^2, x^3)$, then verify whether T is linear or not. 2 Marks

b) Find the matrix of the linear transformation T; $V_2(R) \longrightarrow V_3(R)$ defined by $T(x, y) = (x + y, x, 3x - y)$ with respect to the bases $B_1 = \{(1, 1), (3, 1)\}, B_2 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$. 5 Marks

c) Prove that the range space $R(T)$ and the null space $N(T)$ of the linear transformation $T : V \longrightarrow W$ are subspaces of W & V respectively. 6 Marks

7. a) i) In an inner product space E prove that $\left| \, | \alpha | - | \beta | \, \right| \leq | \alpha - \beta |$ 2 Marks

ii) Prove that if a vector β is orthogonal to $\alpha_1, \alpha_2, ...\alpha_m$ then it is orthogonal to every vector in the subspace spanned by $\alpha_1, \alpha_2, ...\alpha_m$. 2 Marks

b) Find the orthogonal projection of $\beta = (2,1,3)$ on the subspace spanned by $\alpha = (1,0,1)$.

c) If $\alpha = (1,2,3,4)$, $\beta = (0,3,-2,1)$ find a vector of the form $(1,1,0,0) + c_1\alpha + c_2\beta$ orthogonal to both α and β. **6 Marks**

8. a) i) If $Z = \sqrt{x^2+y^2}$ then show that $x\ Z_x + y\ Z_y = Z$ **2 Marks**

ii) Find u_x and u_y if $u = log\frac{x^2+y^2}{\sqrt{xy}}$ **2 Marks**

b) If $U = f(x,y)$ is a homogeneous function of degree n in x and y then Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)\ u$ **5 Marks**

c) Show that a rectangular solid of maximum volume which can be inscribed in a sphere is a cube. **6 Marks**

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