Directorate of Correspondence Course

Third Year B.Sc. Degree Examinations

August /September 2010

(New Scheme)

MATHEMATICS

Paper - IV

Time: 3 hrs.] [Max.Marks : 90

Note: *Answer any SIX of the following questions.*

PART - A

A. *Answer the following.*

1. a) i) Evaluate,

$$\int_C \left[(3x - 2y)\, dx + (y + 2z)dy - x^2\, dz\right]$$

where C is the curve $x = t,\ y = 2t^2,\ z = 3t^3$ and $0 \leq t \leq 1$ — 2 Marks

ii) Evaluate $\int_0^2 \int_0^{x^2} x(x^2 + y^2)\, dy\, dx$ — 2 Marks

b) Let C - be any path leading from the origin to the point $(1, 1, 1)$, then Evaluate $\int_C \left[2xy\, dx + (x^2 + 2yz)\, dy + (y^2 + 1)\, dz\right]$ — 5 Marks

c) Evaluate $\int_R\int xy(x + y)dx\, dy$ over the domain R between $y = x^2$ and $y = x$. — 6 Marks

2. a) i) Evaluate $\int_0^1 \int_0^1 \frac{dx\, dy}{\sqrt{(1-x^2)\,(1-y^2)}}$ — 2 Marks

ii) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dx\, dy\, dz$ — 2 Marks

b) Find the surface area of the cylinder $x^2 + y^2 = 4$ cut off by the cylinder $x^2 + z^2 = 4$. — 5 Marks

c) Find the volume bounded by the surface $z = a^2 - x^2$ and the plane $x = 0,\ y = 0,\ z = 0$ and $z = b$. — 6 Marks

3. a) i) Prove that $\Gamma(n + 1) = n\Gamma(n)$ — 2 Marks

ii) Evaluate $\int_0^{\pi/2} cos^5\theta\ sin^2\theta \cdot d\theta$ — 2 Marks

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b) Show that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n} \frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$ 5 Marks

c) Evaluate $\int_0^a x^4\sqrt{a^2-x^2}\,dx$ 6 Marks

4. a) i) Show that a constant function is Riemann integrable. 2 Marks

ii) Compute $L(p,f)$ and $U(p.f)$ if $f(x) = x$ on $[0,1]$ and $P = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ be a partition of $[0,1]$ 2 Marks

b) Let f - be a bounded function on $[a,b]$ and let P be a partion of $[a,b]$. If P' is a refinement of P, then $L(P,f) \leq L(p',f)$ 5 Marks

c) If $f(x) = x^2$ is defined on $[0,1]$. Show that f is Riemann integrable and $\int_0^1 f(x)\,dx = \frac{1}{3}$ 6 Marks

PART - B

5. a) i) Find the part of the complementary function of

$(1-x)\frac{d^y}{dx^2} + x\frac{dy}{dx} - y = (1-x)^2$ 2 Marks

ii) Verify the condition for exactness of the equation $(1-x^2)y'' - 3xy' - y = 0$ 2 Marks

b) Solve $y'' + (2cos\, x + tan\, x)y' + (cos^2\, x)y = cos^4 x$ by change of independent variable. 5 Marks

c) Solve $y'' - 2tanx\; y' - (a^2+1)y = e^x \cdot sec\, x$ by changing the dependent variable. 6 Marks

6. a) i) Find Wronskion of $y'' + y = sec\, x$. 2 Marks

ii) Write the complementary functions for the cases $P + Qx = 0$ and $a^2 + aP + Q = 0$ 2 Marks

b) Solve, $xy'' - 2(x+1)y' + (x+2)y = (x-2)e^{2x}$ by finding the part of complementary function. 5 Marks

c) Solve $(1-x)y'' + xy' - y = (1-x)^2$ by the method of variation of parameters. 6 Marks

7. a) i) Verify the condition for integrability of the function

$(yz+2x)\,dx + (zx-2z)\,dy + (xy-2y)dz = 0$

ii) Form the partial differential equation by eliminating the arbitrary constants a and b from the equation $z = (x-a)^2 + (y-b)^2$ **2 Marks**

b) Solve, $\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$ **5 Marks**

c) Solve $(bz-cy)p + (cx-az)q = (ay-bx)$ **6 Marks**

8. a) i) If $f(x) = e^{-ax}$ where $-\pi < x < \pi$. Find a_0. **2 Marks**

ii) If $f(x) = x - x^2$ where $-1 < 0 < 1$. Find a_n. **2 Marks**

b) Find the Fourier sereies for

$$f(x) = \begin{cases} -1 & \text{where } -1 < x < 0 \\ 2x & \text{where } 0 < x < 1 \end{cases}$$ **5 Marks**

c) Obtain the Halt range cosine series for the function **6 Marks**

$$f(x) = \begin{cases} x & \text{where } 0 < x < \frac{\pi}{2} \\ \pi - x & \text{where } \frac{\pi}{2} < x < \pi \end{cases}$$

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