

**First Year B.Sc., Degree Examination**

**August / September 2011**

**(Distance Education)**

**Mathematics - Paper I**

Time: 3 hrs]

[Max. Marks: 90

**Note:** Answer any **SIX** full questions of the following choosing at least one from each part.

**PART - A**

1. a) (i) Find  $\phi(100)$  (02)
- (ii) Find g.c.d of 1125 and 1225. (02)
- b) Find the remainder when  $2^{100}$  is divided by 11. (05)
- c) If P is a prime number and  $P/ab$ , then prove that  $P/a$  or  $P/h$ . (06)
2. a) (i) Define Reflexive and symmetric Relations. (02)
- (ii) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^2 + 3x + 1$  and  $g(x) = 2x - 3$ . Find fog and gof. (02)
- b) Let  $f: X \rightarrow Y$  be a function. If C and D are any two subsets of Y then prove that  $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$  (05)
- c) Given  $f: A \rightarrow B$ , defined by  $f(x) = \frac{x+2}{x+3}$ , where  $A = \mathbb{R} - \{3\}$  and  $B = \mathbb{R} - \{+7\}$ . Show that f is Bijective and find  $f^{-1}$ . (06)

**PART - B**

3. a) (i) Evaluate  $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$  (02)
- (ii) If  $4x^2 + 4xy + 5y^2 = 1$  find  $dy/dx$ . (02)
- b) Discuss the continuity of the function defined by
 
$$f(x) = \begin{cases} x^2 + 1 & \text{when } x > 1 \\ 2x & \text{when } x = 1 \text{ at } x = 1 \\ 4x - 2 & \text{when } x < 1 \end{cases}$$
 (05)
- c) If  $y = e^{m \sin^{-1} x}$  then show that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$ . (06)

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4. a) (i) if  $y = a \log \sec \left( \frac{x}{a} \right)$  S.T  $\frac{ds}{dy} = \operatorname{cosec} \left( \frac{x}{a} \right)$ . (02)
- (ii) Find the radius of curvature for the curve  $y = e^x$  at  $x = 0$ . (02)
- b) Show that the pedal equation of the circle  $x^2 + y^2 = 2ax$  is  $r^2 = 2ap$  (05)
- c) Show that the evolute of the curve  $x = a \left[ \cos t + \log \tan \frac{t}{2} \right]$  and  $y = a \sin t$  is  $y = a \cos h \left( \frac{x}{a} \right)$  (06)

**PART - C**

5. a) (i) Find the equation of the plane passing through the point  $(-1, 2, 3)$  and parallel to the plane  $6x + 3y + z + 1 = 0$ . (02)
- (ii) Show that the planes  $6x + 3y + 5z + 1 = 0$  and  $12x + 6y + 10z + 5 = 0$  are parallel. (02)
- b) Find the equation of the plane passing through the points  $(1, 1, 1)$ ,  $(-1, 1, 1)$  and  $(-7, -3, -5)$ . (05)
- c) Determine the mutual positions of the lines  $l_1$  and  $l_2$
- Where  $l_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$  and
- $l_2 : \frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$  (06)
6. a) (i) Find the equation of the sphere whose centre is  $(2, -3, -4)$  and radius is 5. (02)
- (ii) Find the asymptotes parallel to the co-ordinate axes for the curve  $(x-1)y^3 + yx^2 + 3xy = 0$ . (02)
- b) Find all the asymptotes of the curve  $4x^2(y-x) + y(y-2)(x-y) = 4x + 4y - 7$ . (05)
- c) Find the surface area formed by Revolving the curve  $r = a(1 + \cos \theta)$  about its initial axis. (06)

**PART - D**

7. a) (i) Express the Matrix  $A = \begin{bmatrix} 2 & 5 \\ 6 & 7 \end{bmatrix}$  as the sum of symmetric and skew symmetric matrices. (02)
- (ii) Find the eigen values of the matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$  (02)

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b) Find the Rank of the matrix  $\begin{bmatrix} 1 & 6 & 3 & 8 \\ 2 & 4 & 6 & -1 \\ 3 & 10 & 9 & 7 \\ 4 & 16 & 12 & 15 \end{bmatrix}$  (05)

c) Examine the consistency and solve  
 $x - 7y + 15z = -14,$   
 $2x + 3y - 4z = 6$   
 $3x - 4y + 11z = -8$   
 $5x - y + 7z = -2$  (06)

8. a) (i) Evaluate  $\int \frac{\sin x dx}{3 + 4 \cos^2 x}$  (02)

(ii) Evaluate  $\int_0^{\pi/2} \sin^6 x, \cos^2 x, dx$  (02)

b) Evaluate  $\int \frac{2 \cos x - 3 \sin x}{\cos x + 4 \sin x} dx$  (05)

c) Evaluate  $\int_0^{\pi/4} \log(1 + \tan x), dx$  (06)

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