DSB 230

Second Year B.Sc., Degree Examination

August / September 2011

(Distance Education)

Mathematics – Paper II

Time: 3 hrs] *[Max. Marks: 90*

***Note**: Answer any **SIX** full questions of the following choosing at least one from each part.*

PART – A

1. a) (i) Find order and degree of the differential equation

$$\left[1-\left(\frac{dy}{dx}\right)^2\right]^{1/2} = 6.\,\frac{d^2y}{dx^2}$$ (02)

(ii) $y^1 = (x+y)^2$ (02)

b) Solve : $\left[x\cos\frac{y}{x} + y\sin\frac{y}{x}\right]y - y\left[y\sin\frac{y}{x} - x\cos\frac{y}{x}\right]x\frac{dy}{dx} = 0$. (05)

c) Find integrating factor and then solve

$y^2dx + (3xy + y^2 - 1)\ dy = 0$ when $x = 1, y = 2$ (06)

2. a) (i) Solve : $p = \tan(px - y)$. (02)

(ii) Solve : $p^2 + p = 6$. (02)

b) Solve : $x^2p^2 - 2xyp + 2y^2 - x^2 = 0$ (05)

c) Find the orthogonal trajectories of the family

$\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$ λ being a parameter. (06)

PART – B

3. a) (i) Solve : $(D^3 - 7D - 6)\ y = 0$ where $D = \frac{d}{dx}$ (02)

(ii) Solve $(D^2 - a^2)\ y = \sin ax$ where $D = \frac{d}{dx}$. (02)

b) Solve : $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$. where $D = \frac{d}{dx}$ (05)

c) Solve : $(x^3D^3 + 3x^2D^2 + xD + 8)y = 65\cos(\log x)$. (06)

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4. a) (i) Evaluate $\lim_{x \to 0} \frac{\log \sin x}{\cot x}$. (02)

(ii) Evaluate $\lim_{x \to 0} \left[\frac{1}{x} - \frac{\log(1 + x)}{x^2} \right]$ (02)

b) State and prove Roll's theorem. (05)

c) Expand the function $y = \log(1 + \cos x)$ using Maclaurin's series upto the term containing x^4. (06)

PART - C

5. a) (i) If every element of a group G is its own inverse then prove that G is abelian. (02)

(ii) Find all the subgroups of the additive group of all integers modulo 12. (02)

b) Define cyclic group and prove that every subgroup of a cyclic group is cyclic. (05)

c) If G is any finite group and H is any subgroup of G then prove that O(H) divides O(G) (06)

6. a) (i) If $a > b$ and $b > c$ then prove that $a > c$. (02)

(ii) Solve : $|x - 3| < 2$ (02)

b) Find the order of the permutation

$\phi = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 6\,7\,1\,9\,3\,5\,2\,4\,8 \end{pmatrix}$ Also find whether ϕ is even or odd. (05)

c) Obtain the envelope of the family of straight lines $y = cx + a\sqrt{1 + c^2}$. (06)

PART - D

7. a) (i) If $\lim_{n \to \infty} \{x_n\} = l$ and $\lim_{n \to \infty} \{y_n\} = m$ then show that $\lim_{n \to \infty} \{x_n - y_n\} = l - m$. (02)

(ii) Discuss the convergence of the sequence $1 + \frac{1}{3} + \frac{1}{3^2} + \ldots + \frac{1}{3^n}$ (02)

b) Prove that the sequence $\left\{ \frac{2n - 7}{3n + 2} \right\}$ (i) is monotonically increasing. (ii) is bounded (iii) tends to the limit $\frac{2}{3}$ (05)

c) Prove that the sequence $\{a_n\}$ defined by $a_1 = \sqrt{7}$ and $a_{n+1} = \sqrt{7 + a_n}$ converges to the positive root of the equation $x^2 - x - 7 = 0$. (06)

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8. a) (i) Define conditionally convergent series. Give an example (02)

(ii) Discuss the convergence of the series $\sum_{n=0}^{\infty} \frac{\tan^{-1} n}{1+n^2}$ (02)

b) Find the sum to infinity of the series

$$1-\frac{1}{5}+\frac{1.4}{5.10}-\frac{1.4.7}{5.10.15}+\frac{1.4.7.10}{5.10.15.20}-+\ldots\ldots$$ (05)

c) sum to infinity the series

$$\frac{3.5}{1!}x+\frac{4.6}{2!}x^2+\frac{5.7}{3!}x^3+\ldots\ldots$$ (06)

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