DSC 230

Third Year B.Sc., Degree Examination

August / September 2011

(Distance Education)

Mathematics - Paper III

Time: 3 hrs] [Max. Marks: 90

Note: *Answer any **SIX** full questions of the following choosing at least one from each part.*

PART - A

1. a) (i) Prove that every subgroup of an abelian group is a normal subgroup. (02)

(ii) Prove that the mapping $f: (z_1+) \rightarrow G^1 = \{z^a / a \in 2\}$ is a homomorphism (02)

b) Define centre of a group and prove that it is a normal subgroup of G (05)

c) If f : $G \rightarrow G^1$ is an isomorphism of a group G onto a group G^1 and a is any element of G then prove that the order f(a) is equal to the order of a. (06)

2. a) (i) If p is a prime number, then prove that (z_p, t_p, x_p) is a field. (02)

(ii) Let R be a ring & 'a' fixed element of R show that Ra = $\{x \in R / ax = 0\}$ is a sub ring of R. (02)

b) Prove that the Kernel of a homomorphism f: R $\rightarrow R^1$ of rings is an ideal of R. (05)

c) Let f be a homomorphism of a ring R onto the ring R^1 with Kernal K then prove that f(R) is isomorphic to the quotient ring R/K. (06)

3. a) (i) Find the units of Z_5 (02)

(ii) Prove that a polynomial f(x) is divisible by $x - a$ iff f(a) = 0. (02)

b) Show that $x^3 - 9$ is reducible over the field Z_{11} of integers mod 11. (05)

c) If f(x) = $a_0 + a_1x + a_2x^2 + ... + a_nx^n$ is a polynomial in x of degree n with integral coefficients then prove that any rational root of the equation f(x) = 0 must have the form $\frac{r}{s}$ where r / av and s / an. (06)

4. a) (i) Let H be a subgroup of G & K be a normal subgroup of G. Then prove that H∩K is a normal subgroup of H. (02)

(ii) Prove that an ideal of a ring R is a sub ring of R. (02)

b) If f : G → G is a homomorphism of the group G into itself and H is a cyclic subgroup of G. Then prove that f(H) is a cyclic subgroup of G. (05)

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c) Prove that every finite integral domain is a field. (06)

PART - B

5. a) (i) Prove that, in any vector space V, $C\,0 = 0 \; \forall \; C \in F$. (02)

(ii) Prove that $S = \{0, 0, x / x \in R\}$ is a subspace of $V_3(R)$. (02)

b) If S is a subset of a vectorspace V(F), then prove that L(S) is asubspace of V. (05)

c) In $v_3(Z_3)$, find how many vectors are spanned by (1, 2, 1) & (2, 1, 1). Determine all the vectors. (06)

6. a) (i) Prove that the vectors (1, 0, 2), (−1, 0, 1), (0, 4, 2) are linearly independent in $V_3(R)$. (02)

(ii) Show that set containing a single non – zero vector is linearly independent (02)

b) In $V_3(R)$, Show that the plane $x_1 = 0$ is spanned by the vectors (0, 1, 0) & (0, 2, 5) (05)

c) Prove that any linearly independent set of a finite dimensional vector space V is a part of same basis of V. (06)

7. a) (i) If $T : V \to W$ is a linear transformation then prove that $T(0) = 0^1$ (02)

(ii) Find a linear transformation $T : V_2(R) \to V_2(R)$ defined by $T(1, 0) = (1, -1)$ & $T(1, 0) = (0, -2)$ (02)

b) Find the linear transformation for the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$ relative to the basis

$B_1 = \{(1,1,1), (1,2,3), (1,0,0)\}$ & $B_2 = \{(1,1), (1,-1)\}$ (05)

c) Show that the linear transformation $T: V_2(R) \to V_2(R)$ defined by $T(x, y) = (x - y, x - 2y)$ is non – singular and find its inverse. (06)

8. a) (i) If $u = x^y$ find u_x & u_y (02)

(ii) if $u = xy/(x+y)$ show that $xu_x + yu_y = u$ (02)

b) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right), x \neq y$, prove that $xu_x + yu_y = \sin 2u$ (05)

c) Examine $f(x, y) = x^3 + y^3 - 3xy$ for maximum & minimum values (06)

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