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Third Year B.Sc., Degree Examination
Directorate of Distance Education

July / August 2011

MATHEMATICS - Paper - V

Time: 3 hrs.]

[Max.Marks: 90

Note:- Answer any six of the following.

PART - A

1. (a) (i) Verify whether the following points
 $1+i, -2+2i, -2-3i, 6$ are concyclic or not 2 Marks
- (ii) Find whether $\lim_{z \rightarrow 0} \frac{\bar{z}}{z}$ exists or not 2 Marks
- (b) Show that $\arg\left(\frac{2-1+i}{2+i}\right) = \pi/4$ represents a circle find its centre and radius. 5 Marks
- (c) Find the equation of the circle passing through the points $1, i$ and $1+i$.
 Find its centre and radius 6 Marks
2. (a) (i) Show that $f(z) = e^z$ is analytic 2 Marks
- (ii) Prove that an analytic function with constant modulus is constant 2 Marks
- (b) If $f(z)$ is analytic where $f(z) = u + iv$, and $u-v = e^x (\cos y - \sin y)$
 find $f(z)$ in terms of z 5 Marks
- (c) If u & v are harmonic function Show that
 $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)$ is analytic 6 Marks
3. (a) (i) $\int_c (x+2y)dx + (4-2x)dy$ around the ellipse $x = 4 \cos \theta, y = 3 \sin \theta$.
 $0 \leq \theta \leq 2\pi$ taken in anticlockwise direction. 2 Marks
- (ii) Define conformal transformation. Give an example 2 Marks
- (b) If a function $f(z)$ is analytic at all points within and on a closed contour c ,
 Then prove that $\int_c f(z) dz = 0$ 5 Marks
- (c) Prove that the bi linear transformation establishes a one-one
 correspondence from the extended z -plane to the extended w -plane. 6 Marks

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4. (a) (i) Prove that $\Delta - \nabla = \Delta \nabla$ 2 Marks
 (ii) Evaluate $\Delta^2 x^3$ by taking $h = 1$ 2 Marks
 (b) The following table gives the normal weights of babies during first few months of life.

Age in Months	2	5	8	10	12
Weight in Kgs	4.4	6.2	6.7	7.5	8.7

Estimate by Lagrange's method, the normal weight of a baby of 7 months old.

5 Marks

- (c) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 53.5$ from the table.

x	50	51	52	53	54
y	3.6840	3.7084	3.7325	3.7563	3.7798

6 Marks

PART - B

5. (a) (i) Prove that $L \{t^n\} = \frac{n!}{s^{n+1}}$ $n = 0, 1, 2, 3, \dots$ 2 Marks

- (ii) Find $L \left\{ \frac{\cos at - \cos bt}{t} \right\}$ 2 Marks

- (b) Find the laplace transform of the periodic function

$$f(t) = \begin{cases} 1 & \text{for } 0 < t < a/2 \\ -1 & \text{for } a/2 < t < a \end{cases} \text{ and } f(t+a) = f(t) \quad \text{5 Marks}$$

- (c) If $L\{f(t)\} = F(s)$. Then Prove that $L\{u(t-a) f(t-a)\} = e^{-as} F(s)$ where $u(t-a)$ is a unit step function. 6 Marks

6. (a) (i) Find $L \{\sin^3 t\}$ 2 Marks

- (ii) Find $L^{-1} \{ \cot^{-1} s \}$ 2 Marks

- (b) Evaluate $L^{-1} \left[\frac{2s-3}{(s^2+a^2)^2} \right]$ using convolution theorem 5 Marks

- (c) Solve the integral equation problem $f(t) = 4t^2 - \int_0^t f(t-\beta) e^{-\beta} d\beta$ 6 Marks

Contd....3

7. (a) (i) Prove that $f(a+nh) = E^n f(a)$ 2 Marks
 (ii) Find the value of $\Delta \tan^{-1} x$ 2 Marks
- (b) Evaluate $\int_1^3 \frac{1}{x} dx$ by trapezoidal rule by dividing the intervals into 10 equal parts. 5 Marks
 equal parts
- (c) Use Simpsons $\frac{3}{8}$ rule to obtain the approximate value of $\int_0^{0.3} (2x - x^2)^{1/2} dx$ by taking $n = 6$ 6 Marks
8. (a) (i) Use weddle rule to evaluate $\int_{0.2}^{1.4} e^{2x} dx$ given that

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_x	1.4918	2.2255	3.3201	4.9530	7.3890	11.0232	16.4446

2 Marks

- (ii) Show that the real root of $x^3 - 9x + 1 = 0$ lies between 2 and 4 2 Marks
- (b) Find a real root of $x^3 - 2x - 5 = 0$ Newton Raphson method 5 Marks
- (c) If $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ find approximately the values of y (0.2) by taking $h = 0.2$ using Range Kulta method. 6 Marks
