



**First Year B.Sc. Degree Examination, Sept./Oct. 2012**  
**(Directorate of Distance Education)**  
**MATHEMATICS (Paper – I)**

Time : 3 Hours

Max. Marks : 90

**Note:** Answer **any SIX** full questions of the following choosing at least **ONE** from **each** Part.

PART – A

1. a) i) Find the GCD of 24 and 32. 2  
ii) Prove that  $10! \equiv -1 \pmod{11}$ . 2
- b) State and prove Chinese Remainder theorem. 5
- c) Prove that any two nonzero integers  $a$  and  $b$  have a unique positive GCD and it can be expressed in the form  $ma+nb$ , where  $m, n \in \mathbb{Z}$ . 6
2. a) i) Define equivalence Relation and give an example. 2  
ii) Is the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^2 \forall x \in \mathbb{R}$  is one-one ? Justify your answer. 2
- b) Find the equivalence relation induced by the partition  $P = \{(1, 2) (3, 4, 5)\}$  on the set  $A = \{1 2 3 4 5\}$ . 5
- c) Prove that the set of all rational numbers  $\mathbb{Q}$  is countable. 6

PART – B

3. a) i) Evaluate  $\lim_{x \rightarrow 0} x \sin (1/x)$ . 2  
ii) Find the  $n^{\text{th}}$  derivative of  $y = e^x \cos x$ . 2
- b) Discuss the differentiability of  $f(x) = |x| + |x-1|$  at  $x = 0$  and  $x = 1$ . 5
- c) If  $x = \sin t$  and  $y = \cos mt$ , prove that  $(1 - x^2) y_{n+2} - (2n + 1) xy_{n+1} + (m^2 - n^2) y_n = 0$ . 6

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4. a) i) Find  $\frac{ds}{dt}$  for the curve  $x = a(t + \sin t)$  and  $y = a(1 - \cos t)$ . 2
- ii) Show that the radius of curvature of the curve  $pa^2 = r^3$  is  $a^2/3r$ . 2
- b) Show that the pair of curves  $r^n = a^n \cos n\theta$  and  $r^n = b^n \sin n\theta$  intersect each other orthogonally. 5
- c) Find the evolute of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ . 6

PART – C

5. a) i) Find the parametric representation of the line through the points  $(1, -1, 1)$  and  $(2, 3, 0)$ . 2
- ii) Verify whether the following planes are parallel.  
 $x + y + z - 3 = 0$   
 $2x + 7y + 3z - 5 = 0$  2
- b) Find the equation of the plane through the point  $(4, -1, 0)$  and the line  $x = t$ ,  $y = 2t$ ,  $z = 3t$ . 5
- c) Determine the mutual position of lines  $l_1$  and  $l_2$ .  
 $l_1 : x = 2 - t, y = 1 - t, z = 1 - 3t$   
 $l_2 : x = 3 - 2s, y = 4 - s, z = 12 - 2s$ . 6
6. a) i) Find the centre and radius of the sphere  $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$ . 2
- ii) Find the asymptotes parallel to the co-ordinate axes for the curve  $x^2y^2 = a^2(x^2 + y^2)$ . 2
- b) Find all the asymptotes of the curve  $3x^3 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0$ . 5
- c) Find the surface area of the solid generated by the revolution of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$  6



PART – D

7. a) i) Express the matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  as the sum of symmetric and skew symmetric matrices. 2

ii) Find the Rank of the matrix  $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$ . 2

b) Find the inverse of the matrix A by the elementary transformations

$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ . 5

c) Find the non-trivial solution of the system

$x + 3y - 2z = 0$   
 $2x - y + 4z = 0$   
 $x - 11y + 14z = 0$ . 6

8. a) i) Evaluate  $\int \frac{dx}{1 + \cos x}$  2

ii) Evaluate  $\int e^x \left[ \frac{1 + \sin x}{1 + \cos x} \right] dx$  2

b) Evaluate  $\int \frac{dx}{x^2 \sqrt{x^2 + 1}}$  5

c) Evaluate  $\int_0^1 \frac{\cos x}{\sqrt{1 - x^2}} dx$ . 6