DSC – 231

Third Year B.Sc. Degree Examination, Sept./Oct. 2012
Directorate of Distance Education
MATHEMATICS – IV

Time : 3 Hours Max. Marks : 90

***Note** : Answer **any six** of the following.*

PART – A

1. a) i) Evaluate $\int_c \left[\left(2y + x^2\right)dx + \left(3x - y\right)\right]$, where c is the curve given by x = 2t, $y = t^2 + 3,\ 0 \le t \le 1$. **2**

   ii) Evaluate $\int_1^2 \int_3^4 \left(xy + e^y\right) dx\, dy$ **2**

   b) Evaluate $\int_c \left[\left(x + 2y\right) dx + \left(4 - 2x\right) dy\right]$ around the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ in the countes clockwise. **5**

   c) Let C be any path leading from the origin to the point (1, 1, 1). Show that $\int_c \left[2\, xydx + \left(x^2 + 2yz\right) dy + \left(y^2 + 1\right) dz\right] = 3$ **6**

2. a) i) Evaluate $\int_0^1 \int_0^{y^2} e^{x/y}\ dxdy$ **2**

   ii) Evaluate $\int_2^3 \int_2^1 \int_1^2 \frac{1}{xyz}\ dx\, dy\, dz$ **2**

   b) Find the area of the surface $y^2 + z^2 = 2x$, cut off by the plane x = 1. **5**

   c) Find the volume of the region above the xy plane bounded by the paroled $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = a^2$. **6**

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3. a) i) Evaluate $\int_0^\infty x^5 \ e^{-x} \ dx$ **2**

ii) Prove that β (M, n) = β(n, M) **2**

b) Show that $\int_0^a x^4 \sqrt{a^2 - x^2} \ dx = \frac{\pi a^6}{32}$ **5**

c) Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)\,m+n}$ hence find the value of

$\int_0^\infty \frac{x^3(1-x^8)}{(1+x)^{14}} dx$ **6**

4. a) i) If f : [a, b] $\rightarrow$R is bounded function, then prove that L (p, f) $\leq$ U (p, f), where p is any partition of [a, b]. **2**

ii) State the fundamental theorem of integral calculus. **2**

b) Evaluate $\int_{-1}^{1} |x| \ dx$ **5**

c) State and prove Darboux theorem. **6**

PART – B

5. a) i) Find the part of complimentary function of

$xy'' - 2(x+1)\,y' + (x+2)\,y = (x-2)\,e^{2x}$ **2**

ii) Find the w wronskian for the equation $y'' - y = \frac{2}{1+e^x}$ **2**

b) Solve $(1+x^2)^2 \frac{d^2y}{dx^2} + 2x\,(1+x^2)\frac{dy}{dx} + y = 0$ using the transformation

z = tan–1x **5**

c) Solve $y'' - (2 \ \tan x)y' + 5y = e^x \sec x$ by changing the dependent variable. **6**

6. a) i) Verify the condition of exactness of the equation $x\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = \cos x$ **2**

ii) Write the complementary functions for the cases 2P + Q + 4 = 0 and P + Qx = 0. **2**

b) Verify for exactness and solve the equation

$$(\sin x)\frac{d^2y}{dx^2} + (\cos x)\frac{dy}{dx} + (2\sin x)y = 0$$ **5**

c) Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by the method of variation of parameters. **6**

7. a) i) Verify the condition for integrability of the equation yz dx + 2 zx dy – 3 xydz = 0. **2**

ii) Form the partial differential equation by eliminating constrants a and b in z = (x + a) (y + b). **2**

b) Find general and singular solution of $z = px + qy + \sqrt{\alpha p^2 + \beta q^2 + \gamma}$ where α, β, γ are constants. **5**

c) Solve (y – z) p + (z – x) q = x – y. **6**

8. a) i) If f(x) = x – 1 in $-\pi \le x \le \pi$ find a_0. **2**

ii) Express f(x) = $\frac{1}{x+3}$ is the sum of an even and odd functions **2**

b) Find the half range cosine series of the function f(x) = (x – 1)2 in (0, 1). **5**

c) Find the half Fouries series of the function f(x) = x – x^2 over the internal (–1, 1) and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + ---$ **6**