DSC – 232

Third Year B.Sc. Degree Examination, Sept./Oct. 2012
(Directorate of Distance Education)
MATHEMATICS (Paper – V)

Time : 3 Hours Max. Marks : 90

Note : *Answer **any SIX** of the following.*

PART – A

1. a) i) Find the real and imaginary parts of Exp $\left(5+i\frac{\pi}{2}\right)$. **2**

ii) Evaluate $\lim_{z\to e^{i\frac{\pi}{4}}}\left[\frac{z^2}{z^4+z^2+1}\right]$. **2**

b) If $\frac{z-i}{z-1}$ is purely imaginary, then show that its locus is a circle and find its centre and radius. **5**

c) Find the equation of the circle passes through the points 1 – i, 2i, 1 + i and find its centre and radius. **6**

2. a) i) Verify f(z) = $\bar{z}$ is analytic or not. **2**

ii) If f(z) is analytic in an open set S and $f'(z)=0\ \forall\ z\in S$ then show that f(z) is a constant. **2**

b) If f(z) = u + iv is analytic and u – v = (x – y) (x^2 + 4x + y^2). Find f(z) in terms of z. **5**

c) If f(z) = u + iv is analytic show that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2=4|f'(z)|^2$. **6**

P.T.O.

3. a) i) Evaluate $\int_{(0,1)}^{(3,10)} (3x+y)\,dx + (2y-x)dy$ along the curve $y = x^2 + 1$. **2**

ii) Find the fixed point of the bilinear transformation $w = \frac{1-z}{1+z}$. **2**

b) State and prove Cauchy's integral formula. **5**

c) Discuss the transformation $w = e^z$. **6**

4. a) i) Prove that $\nabla = 1 - E^{-1}$. **2**

ii) Evaluate $\Delta^2 \sin x$. **2**

b) From the following table find the number of students obtain less than 45 marks. **5**

No. of Students	31	42	51	35	31
Marks	30 to 40	40 to 50	50 to 60	60 to 70	70 to 80

c) Find $f'(x)$ and $f''(x)$ of the function f(x) at x = 1.5 given **6**

x	1.5	2	2.5	3	3.5
f(x)	3.375	7	13.625	24	38.875

PART – B

5. a) i) Find L [cos h mt]. **2**

ii) Find L [2 sin t. sin 3t]. **2**

b) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} 1 & \text{for } 0 < t < \frac{a}{2} \\ -1 & \text{for } \frac{a}{2} < t < a \end{cases} \text{ and } f(t+a) = f(a).$$ **5**

c) Express $f(t) = \begin{cases} t^2 & 0 < t < 2 \\ 6 & t > 2 \end{cases}$ as a unit step function and also find Laplace transform. **6**

6. a) i) Evaluate $L[e^{2t}(2t^2 - 3t + 4)]$. **2**

ii) Find $L^{-1}[\text{Cot}^{-1}(s + 1)]$. **2**

b) Verify the convolution theorem for the functions f(t) = sin t and g(t) = cost. **5**

c) Solve by using Laplace transform $y'' - 9y = -8e^{t}$, given y(0) = 0 and $y'(0) = 10$. **6**

7. a) i) Prove that $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x} = e^x$ taking h as the interval of difference. **2**

ii) Evaluate $\Delta \cot(2^x)$. **2**

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using Trapezoidal rule with h = 0.2. **5**

c) Use Simpson $\frac{3^{th}}{8}$ rule to obtain the approximate value of $\int_0^{0.3} (2x - x^2)^{1/2}\, dx$ by taking n = 6. **6**

8. a) i) Using Weddle's Rule Evaluate $\int_0^1 \frac{1}{1+x}dx$ given that

x	0	1/6	2/6	3/6	4/6	5/6	6/6
y	1	6/7	6/8	6/9	6/10	6/11	6/12

. **2**

ii) Show that the real root of $x^3 - 9x + 1$ lies between 2 an 4. **2**

b) Using Newton Raphson method, find the real root of the equation $xe^x - 2 = 0$ correct to three significant figures. **5**

c) If $\frac{dy}{dx} = x + y$ and y = 1 when x = 0. Find approximate value of y when x = 0.2 using Runge Kutta method. **6**
