Q.P. Code – 50824

Third Year B.Sc. Degree Examination

SEPTEMBER/OCTOBER 2013

(Directorate of Distance Education)

(DSC 231) Paper IV – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX**, choosing atleast **TWO** questions from each Part.*

PART – A

1. (a) (i) Evaluate $\int_C (3x+y)dx+(2y-x)dy$ along the curve $y=x^2+1$ from (0, 1) to (3, 10) **2**

(ii) Evaluate $\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$. **2**

(b) Evaluate $\int_C (x+y+z)dS$ where C is the line joining the points (1, 2, 3) and (4, 5, 6). **5**

(c) Show that $\int_C (4xy-3x^2z^2)dx+2x^2dy-2x^3z\,dz$ is independent of the path joining from $(3,-1,1)$ to $(0,1,2)$ and hence evaluate. **6**

2. (a) (i) Evaluate $\int_0^2\int_0^{x^2} x(x^2+y^2)dy\,dx$. **2**

(ii) Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,dx\,dy\,dz$. **2**

(b) Find the surface area of the cylinder $x^2+y^2=4$ cut by the cylinder $x^2+z^2=4$. **5**

(c) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **6**

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3. (a) (i) Evaluate $\int_0^\infty e^{-4x} x^{3/2}\, dx$. **2**

(ii) Prove that $\beta(m, n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\, dy$. **2**

(b) Evaluate $\int_0^a x^4 \sqrt{a^2 - x^2}\, dx$. **5**

(c) Show that $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx = \beta(m, n)$. **6**

4. (a) (i) Define Lower Riemann integral and upper Riemann integral of f over $[a, b]$. **2**

(ii) State the criterion for the condition of integrability. **2**

(b) State and prove Darboux Theorem. **5**

(c) Show that the function $f(x) = x^2$ is integrable over $[a\ b]$ and hence show that $\int_a^b x^2 dx = \frac{b^3 - a^3}{3}$. **6**

PART – B

5. (a) (i) Find the part of complimentary function of

$$xy'' + 2(x+1)y' + (x+2)y = (x+2)e^{2x}.$$ **2**

(ii) Find the Wronskian W for the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos\left(e^{-x}\right).$$ **2**

(b) Solve $x^2y'' + xy' - y = 2x^2\ (x > 0)$ given that $\frac{1}{x}$ is a part of complimentary function. **5**

(c) Solve $\frac{d^2y}{dx^2} + (2\cos x + \tan x)\frac{dy}{dx} + \left(\cos^2 x\right)y = \cos^4 x$ by changing the independent variable. **6**

6. (a) (i) Show that $\cos x\, y'' + 2\sin x\, y' + 3\cos x\, y = \tan^2 x$ is exact. **2**

(ii) Solve $\dfrac{dx}{z^2 y} = \dfrac{dy}{z^2 x} = \dfrac{dz}{x y^2}$. **2**

(b) Solve $(1-x)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = (1-x)^2$ where $x \neq 1$ by the method of variation of parameters. **5**

(c) Show that $x^2(1+x)\dfrac{d^2y}{dx^2} + 2x(2+3x)\dfrac{dy}{dx} + 2(1+3x)y = 0$ is exact and solve it. **6**

7. (a) (i) Verify the condition for integrability for $(yz+2x)dx + (zx-2z)dy + (xy-2y)dz = 0$. **2**

(ii) Form the partial differential equation by eliminating the arbitrary constants a and b from the equation $(x-a)^2 + (y-b)^2 + z^2 = K^2$. **2**

(b) Verify the condition for integrability and solve $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$. **5**

(c) Solve $(y^3x - 2x^4)p + (2y^4 - x^3y)q = q\,z(x^3 - y^3)$. **6**

8. (a) (i) Find the Fourier coefficient a_0 for the function $f(x) = e^{-ax}$ in the interval $-\pi < x < \pi$. **2**

(ii) Obtain the Half range sine series for the function $f(x) = x(\pi - x)$ over the interval $(0, \pi)$. **2**

(b) Find the Fourier series of the function

$$f(x) = \begin{cases} 1 & \text{for } -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ -1 & \text{for } \dfrac{\pi}{2} < x < \dfrac{3\pi}{2} \end{cases} \text{ and } f(x+2\pi) = f(x).$$ **5**

(c) Expand the function $f(x) = \begin{cases} 1+2x & \text{in } -3 < x \le 0 \\ 1-2x & \text{in } 0 \le x < 3 \end{cases}$ as a Fourier series and deduce that $\dfrac{\pi^2}{8} = \displaystyle\sum_{n=1}^{\infty} \dfrac{1}{(2n-1)^2}$. **6**
