Q.P. Code – 50824

Third Year B.Sc. Degree Examination

OCTOBER/NOVEMBER 2014

(Directorate of Distance Education)

(DSC 231) Paper IV – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX** choosing atleast TWO questions from each Part.*

PART – A

1. (a) (i) Evaluate $\int_C xy\,dx$, where C is the arc of the parabola $x = y^2$ from $(1, -1)$ to $(1, 1)$. **2**

(ii) Evaluate $\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\,\sqrt{1-y^2}}$. **2**

(b) Show that $\int_C y\,dx - x\,dy = 24\pi$, where C is the arc of the cycloid $x = 2(t-\sin t),\ y = 2(1-\cos t)$ joining the points (0, 0) and $(4\pi, 0)$. **5**

(c) Evaluate $\iint_R xy(x+y)\,dx\,dy$ over the region R bounded between the parabola $y = x^2$ and the line $y = x$. **6**

2. (a) (i) Evaluate $\int_0^1\int_0^{x^2} e^{\frac{y}{x}}\,dy\,dx$. **2**

(ii) Evaluate $\int_0^1\int_0^2\int_1^2 xyz^2\,dx\,dy\,dz$. **2**

(b) Find the area of the surface $z = \sqrt{x^2+y^2}$, $\frac{1}{16} < x^2 + y^2 < \frac{1}{4}$. **5**

(c) Find the volume of the region above the xy plane bounded by the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = a^2$. **6**

3. (a) (i) Prove that $\overline{|n+1} = n!$. **2**

(ii) Show that $\int_0^1 x^5 \left(1-x^3\right)^{10} dx = \frac{1}{396}$. **2**

(b) Prove that $\beta(m, n) = \frac{\overline{|m}\,\overline{|n}}{\overline{|m+n}}$. **5**

(c) Show that $\int_0^2 x\left(8-x^3\right)^{1/3} dx = \frac{16\pi}{9\sqrt{3}}$. **6**

4. (a) (i) Define Refinement and common refinement of a partition. **2**

(ii) If $f:[a, b] \to R$ is bounded function, then prove that $L(P, f) \le U(P, f)$ where P is any partition of $[a, b]$. **2**

(b) If f and g are integrable functions over $[a, b]$, then prove that fg is also integrable over $[a, b]$. **5**

(c) Prove that a bounded function f is Riemann integrable over $[a, b]$ iff $\forall \in > 0 \;\; \exists$ a partition P of $[a, b]$ with $\|P\| < S$ such that

$$\left|\sum_{i=1}^{n} f(t_i) f_i - \int_a^b f\,dx\right| < \in .$$ **6**

PART – B

5. (a) (i) Find the part of complimentary function of

$$(x\sin x + \cos x)\frac{d^2y}{dx^2} - x\cos x\frac{dy}{dx} + y\cos x = 0.$$ **2**

(ii) Find the Wronskian W for the equation $y'' + y = \tan x$. **2**

(b) Solve $xy'' - y' + 4x^3 y = 4x^3 e^{x^2}$ by changing the independent variable. **5**

(c) Solve $y'' - 2\tan x \cdot y' + 5y = e^x \sec x$ by changing the dependent variable. **6**

6. (a) (i) Verify the condition of exactness of the equation

$$\cos x\frac{d^2y}{dx^2}+2\sin x\frac{dy}{dx}+3y\cos x=\tan^2 x.$$ **2**

(ii) Write the complementary function for the cases $1+P+Q=0$ and $P+Qx=0$. **2**

(b) Solve $\frac{d^2y}{dx^2}+(\cot x)\frac{dy}{dx}-\left(\operatorname{cosec}^2 x\right)y=0$, given that $\cot x$ is a part of complementary function. **5**

(c) Solve $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$ by the method of variation of parameters. **6**

7. (a) (i) Verify the condition for integrability of the equation

$$z^2\,dx+\left(z^2-2yz\right)dy+\left(2y^2-yz-zx\right)dz=0.$$ **2**

(ii) Form the partial differential equation by eliminating the constants a and b in $z=(x-a)^2+(y-b)^2$. **2**

(b) Solve $\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$. **5**

(c) Solve $\left(p^2+q^2\right)\left(x^2+y^2\right)=1$. **6**

8. (a) (i) Find the Fourier coefficient a_0 for $f(x)=x^2$, $-\pi\le x\le\pi$. **2**

(ii) Express $f(x)=e^x$ as the sum of even and odd functions. **2**

(b) Find the half range cosine series of the function $f(x)=\pi-x$ in $(0,\pi)$. **5**

(c) Find the Fourier series of the function $f(x)=x-x^2$ over the interval $(-1,1)$. **6**
