Third Year B.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

Paper –III: DSC 230: MATHEMATICS

Time: 3hrs.] *[Max. Marks: 90*

Note: Answer any SIX of the following:

PART – A

1. a) i) Define normal subgroup with an example.

   ii) Find the quotient group of $G = \{1, -1, i, -i\}$ by the subgroup $H = \{1, -1\}$ under multiplication. (2 +2)

   b) Prove that a subgroup N of a group G is normal subgroup of G if and only if every right coset of N in G is a left coset of N in G. (5)

   c) State and prove the fundamental theorem on homomorphism of groups. (6)

2. a) i) Define an integral domain with an example.

   ii) Show that the unity element of a sub ring S of a ring R with unit element may be different from the unity of R. (2 + 2)

   b) Show that the only ideals of a field F are $\{0\}$ and F. (5)

   c) Find all the principal ideals of the ring $(Z_8, +_8, X_8)$ (6)

3. a) i) Show that $3+\sqrt{5}$ and $1-\sqrt{5}$ are associates in $Z[\sqrt{5}]$.

   ii) Factorize $x^4 + 4$ into linear factors over Z_5. (2 + 2)

   b) Find the gcd of $x^4 + x^3 + 4x^2 + 4x - 2$ and $x^3 + x^2 + 5x - 2$ in Z_7. (5)

   c) If $P(x) = a_0x^n + a_1x^{n-1} + ... + a_n$ be a polynomial with integral co-efficients, then prove that any rational root of $p(x) = 0$ must have the form $\frac{r}{s}$, where r/a_n and s/a_0. (6)

4. a) i) Prove that every subgroup of an abelian group is normal subgroup.

   ii) Is Z_5 is a field? Why? (2 + 2)

b) If $f: R \to R^{+}$ is defined by $f(x)=e^{x}$, then prove that *f* is an isomorphism. Find its kernel, where R is the additive group of real numbers and R^{+} is the multiplicative group of positive real numbers. (5)

c) If $f: R \to R^{1}$ be a homomorphism of rings R on to R^{1} with kernel K, then prove that *f* is one-one if and only if $K=\{0\}$. (6)

PART – B

5. a) i) In a vector space *V*, over the field *F*, If $C_1\alpha = C_2\alpha$ and $\alpha \neq 0$ then show that $C_1 = C_2$

ii) Give an example to show that the union of two subspaces of a vector space *V* need not be a subspace of *V*. (2 + 2)

b) In the vector space $V_3(R)$, let $\alpha=(1,2,1)$, $\beta=(3,1,5)$ and $\gamma=(-1,3,-3)$. Show that the subspace spanned by (α, β) and (α, β, γ) are the same. (5)

c) If *n* vectors span a vector space $V(F)$ and *r* vectors of *v* are linearly independent, then prove that $n \geq r$. (6)

6. a) i) Determine whether the vectors $(1,0,1)$, $(0,2,2)$, $(3,7,1)$ of $V_3(R)$ linearly dependent or linearly independent.

ii) In an '*n*' dimensional vector space $V(F)$ any *n* + 1 elements of V are linearly dependent. (2 + 2)

b) Prove that any two bases of a finite dimensional vector space V have the same finite number of elements. (5)

c) Find the basis and dimension of the subspace spanned by the vectors $(2, 4, 2)$, $(1, -1, 0)$, $(1, 2, 1)$ and $(0, 3, 1)$ in $V_3(R)$. (6)

7. a) i) If $T: V_1(R) \to V_3(R)$ is defined by $T(x)=(x, x^2, x^3)$ verify T is linear or not.

ii) Find the matrix of the linear transformation $T: V_3(R) \to V_2(R)$ defined by $T(x, y, z)=(x+y, y+z)$ relative to bases

$B_1=\{(1,1,1), (1, 0, 0), (1, 1, 0)\}$ of $V_3(R)$

$B_2=\{e_1, e_2\}$ of $V_2(R)$. (standard basis of $V_2(R)$ (2 + 2)

b) Find the range, null space, rank and nullity of the linear transformation $T: V_3(R) \to V_2(R)$ defined by $T(x, y, z)=(y-x, y-z)$ and also verify rank-nullity theorem. (5)

c) Find the orthonormal basis for a subspace of a Euclidian space $(2, 0, 0, 0)$, $(1, 3, 3, 0)$, $(0, 4, 6, 1)$ (6)

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8. a) i) If $f(x,\ y) = x^y + y^x$ then find f_x and f_y.

ii) If $u = \phi(y + ax) + \psi(y - ax)$ then show that $\dfrac{\partial^2 u}{\partial x^2} = a^2 . \dfrac{\partial^2 u}{\partial y^2}$ (2 + 2)

b) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$ where $x \neq y$, then prove that $x.\dfrac{\partial u}{\partial x} + y.\dfrac{\partial u}{\partial x} = \sin 2u$ (5)

c) Investigate the maximum and minimum of the function $f(x, y) = 2x^2 - xy + y^2 + 7x$ (6)

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