Third Year B.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

Paper –V: DSC 232: MATHEMATICS

Time: 3hrs.] *[Max. Marks: 90*

Note: Answer any SIX of the following:

PART – A

1. a) i) Find the real and imaginary parts of e^z.

   ii) Evaluate $\lim_{z\to i} \dfrac{z^2+1}{z^6+1}$ (2 + 2)

   b) Show that $\left|\dfrac{z-2}{z+2}\right| = 3$ represents a circle. Find its centre and radius. (5)

   c) Find the equation of the circle passing through the points $1-i,\ 2i, 1+i$. (6)

2. a) i) Verify C – R equations for the function $f(z) = \cos z$.

   ii) Show that $u = \sinh x \cos y$ is harmonic (2 + 2)

   b) Find the analytic function whose imaginary part is $e^x (x \sin y + y \cos y)$. (5)

   c) If $f(z) = u + iV$ is analytic then show that $\left[\dfrac{\partial |f(z)|}{\partial x}\right]^2 + \left[\dfrac{\partial |f(z)|}{\partial y}\right]^2 = |f^1(z)|^2$. (6)

3. a) i) Evaluate $\int_C \dfrac{e^z}{z^2}\, dz$ where C is $|z| = 1$.

   ii) Find the fixed point of $w = \dfrac{2z-1}{z}$ (2 + 2)

   b) State and prove Cauchy's integral formula. (5)

   c) Show that the transformation $w = z^2$ transform the lines parallel to co-ordinate axes into a set of confocal parabolas in $w-$plane. (6)

4. a) i) Prove that $\Delta - \nabla = \Delta\nabla$

   ii) Prove that $E = e^{hD}$ where E is the shift operator, D is the differential operator and h is the interval of differencing. (2 + 2)

b) From the following table find the number of students who secured marks not more than 45 given that (5)

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No of Students	35	48	70	40	22

c) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.6$ from the following table (6)

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$f(x)$	7.989	8.403	8.781	9.129	9.451	9.750	10.031

PART – B

5. a) i) Find $L[2\sin 3t + 4\cos 2t]$

ii) Find $L[\cosh 4t \, \sin 3t]$ (2 + 2)

b) Find the Laplace transform of periodic function $f(t) = \begin{cases} E & for \quad 0 \le t < \frac{T}{2} \\ -E & for \quad \frac{T}{2} \le t \le T \end{cases}$ and

$f(t+T) = f(t)$ (5)

c) If $f(t)$ is continuous for $t \ge 0$. Find $L^{-1}\left[\frac{(1-e^{-2s})(1-3e^{-2s})}{8^2}\right]$ and evaluate $F(1)$, $F(3)$ and $F(5)$. (6)

6. a) i) Find $L[t\cos at]$

ii) Find $L^{-1}\left[\frac{s+1}{s^2+2s-8}\right]$ (2 + 2)

b) Using convolution theorem find $L^{-1}\left[\frac{S}{(S^2+a^2)^2}\right]$ (5)

c) Solve by using Laplace transforms $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$ given $y(0) = 0$ & $\frac{dy}{dx} = 0$ when $t = 0$ (6)

7. a) i) Evaluate $\Delta[x(x+2)]$ by taking $h = 1$

ii) Show that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$ (2 + 2)

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b) Evaluate $\int_0^1 e^x \, dx$ approximately in steps of 0.2 using Trapezoidal rule. (5)

c) Using Simpson's $3/8^{th}$ rule evaluate $\int_0^1 \frac{x}{1+x^2} \, dx$ by dividing the interval into 3 equal parts. (6)

8. a) i) Evaluate $\int_0^6 y_x \, dx$ using Weedle's rule from the following data

x	0	1	2	3	4	5	6
y_x	1	0.5	0.2	0.1	0.0588	0.0385	0.027

ii) Using Picard's method of successive approximation find first approximation of $\frac{dy}{dx} = x + y$ given $y(0) = 1$. (2 + 2)

b) Using modified Euler's method find an approximate value of y for $x = 0(0,2)\ 0.4$ for $\frac{dy}{dx} = x + y.$ given $y = 1$ when $x = 0$. (5)

c) Solve $\frac{dy}{dx} = xy$ for $x = 1.2$ given that $y(1) = 2$ by Runge – Kutta method. (take $h = 0.2$) (6)

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